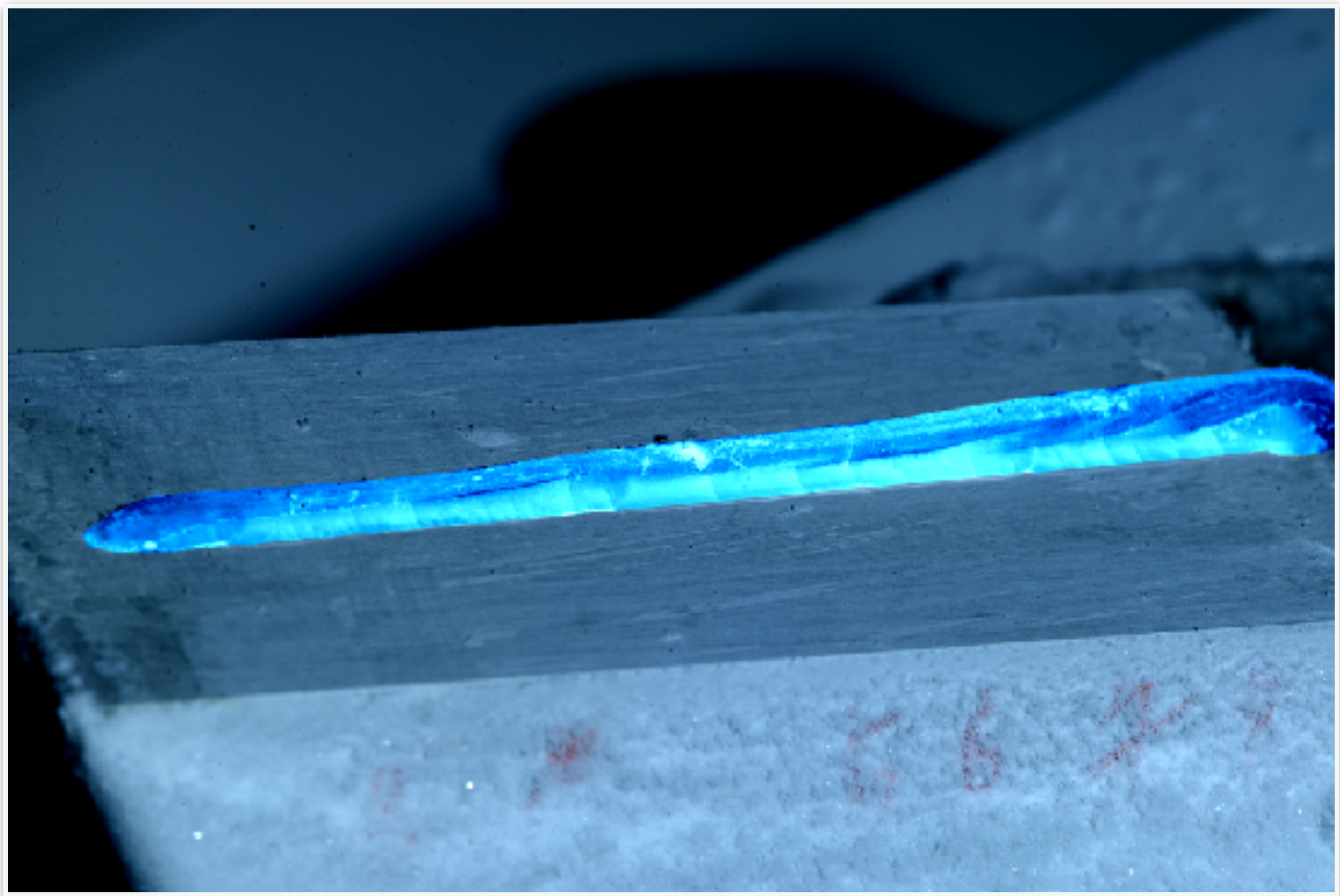
FREEZING A RIVULET



Axel Huerre, Christophe Josserand
(LadHyX, École Polytechnique, Palaiseau, France)

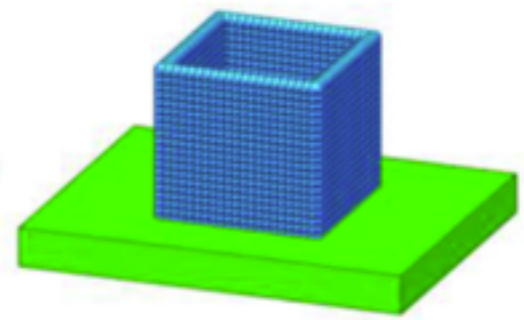
Antoine Monier, Thomas Séon
(Institut ∂'Alembert, CNRS & UPMC, Paris, France)

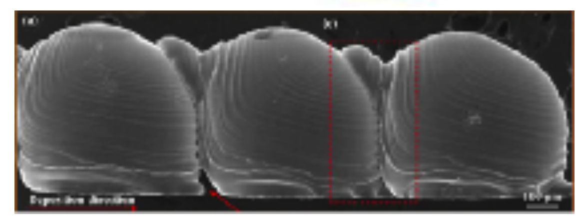
CAPILLARY FLOW AND SOLIDIFICATION

Natural flow

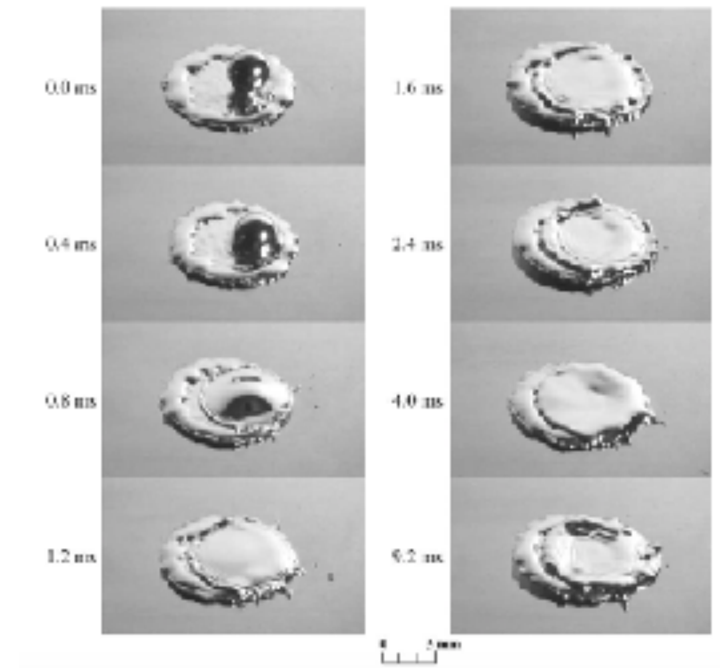
Ice cascade, icicle, ice stalagmite …

Metallurgy (coating, 3D printing …)



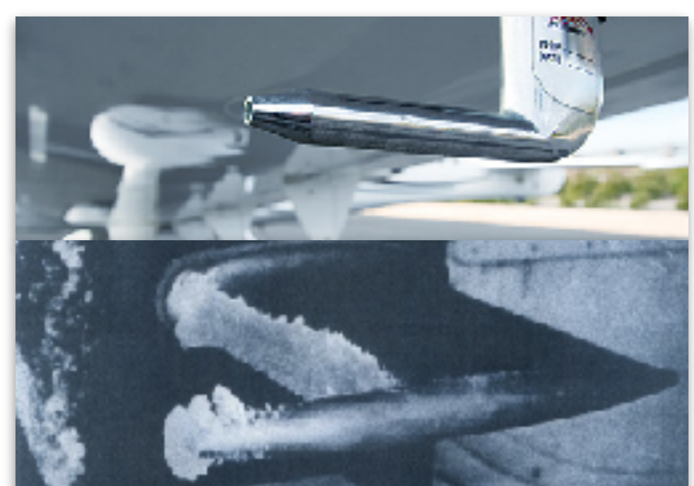


Uniform aluminum droplet deposition manufacturing
Yi et al, *IJMTM* (2018)



Ghafouri-Azar et al, *IJHMF* (2003)

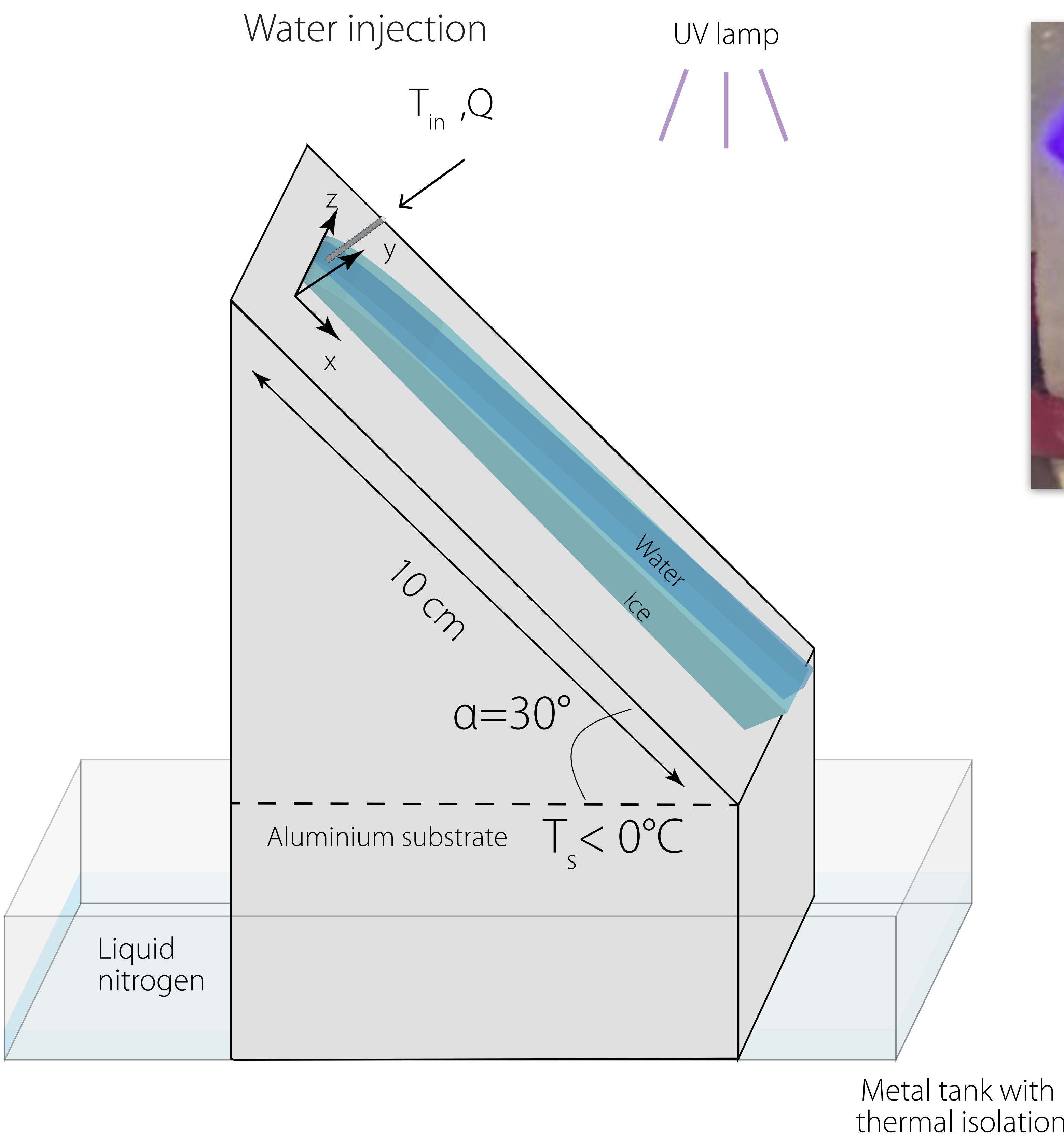
Aircraft icing

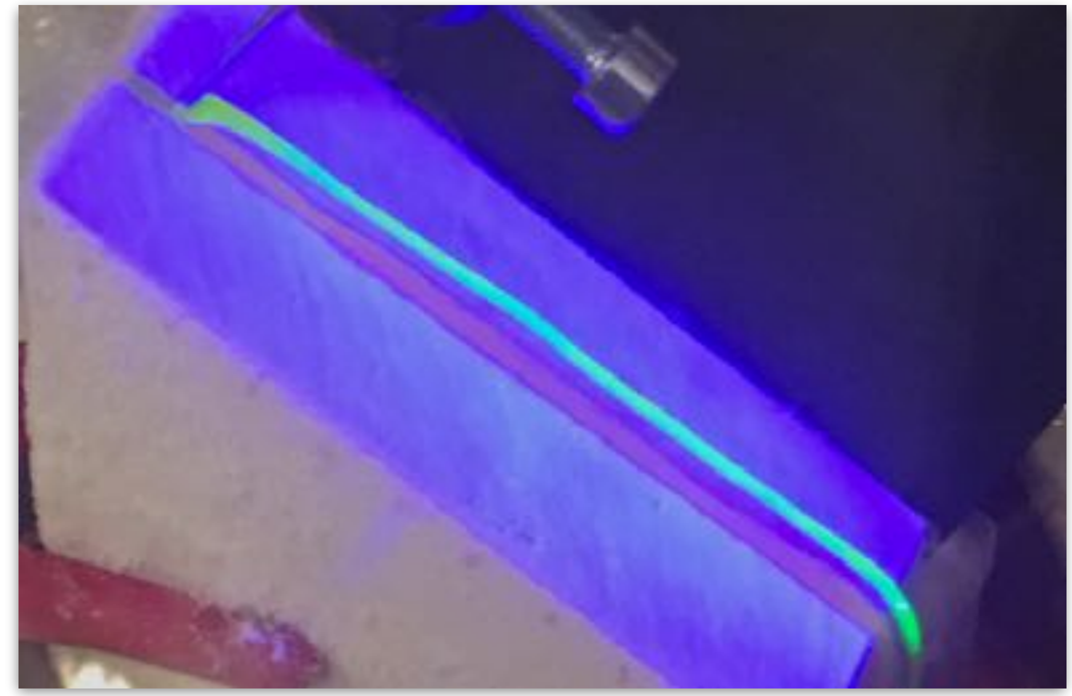


Pitot sensors freezing

Crash Rio-Paris (2009)
Crash in Russia (2018)

EXPERIMENTAL SETUP

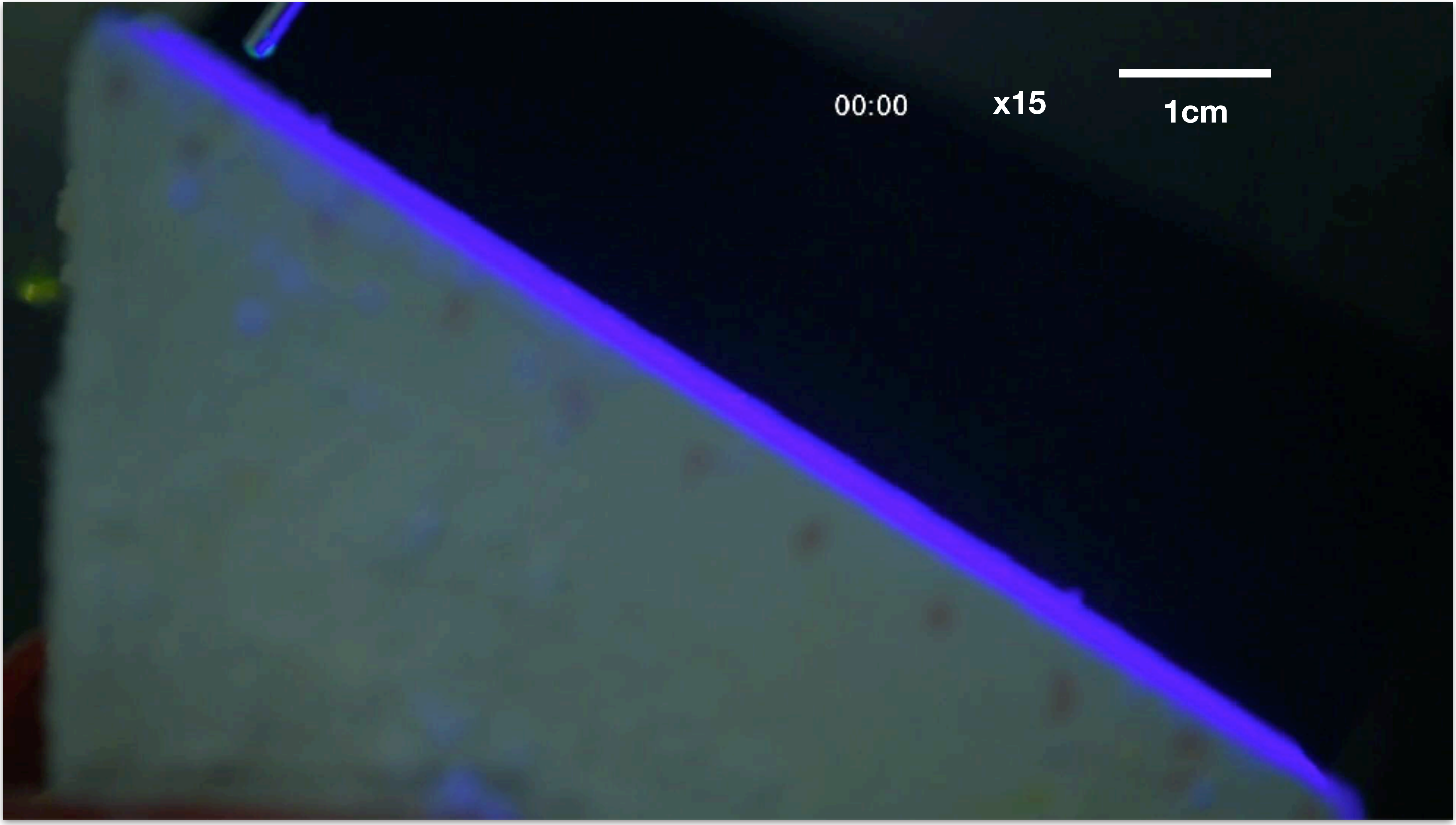




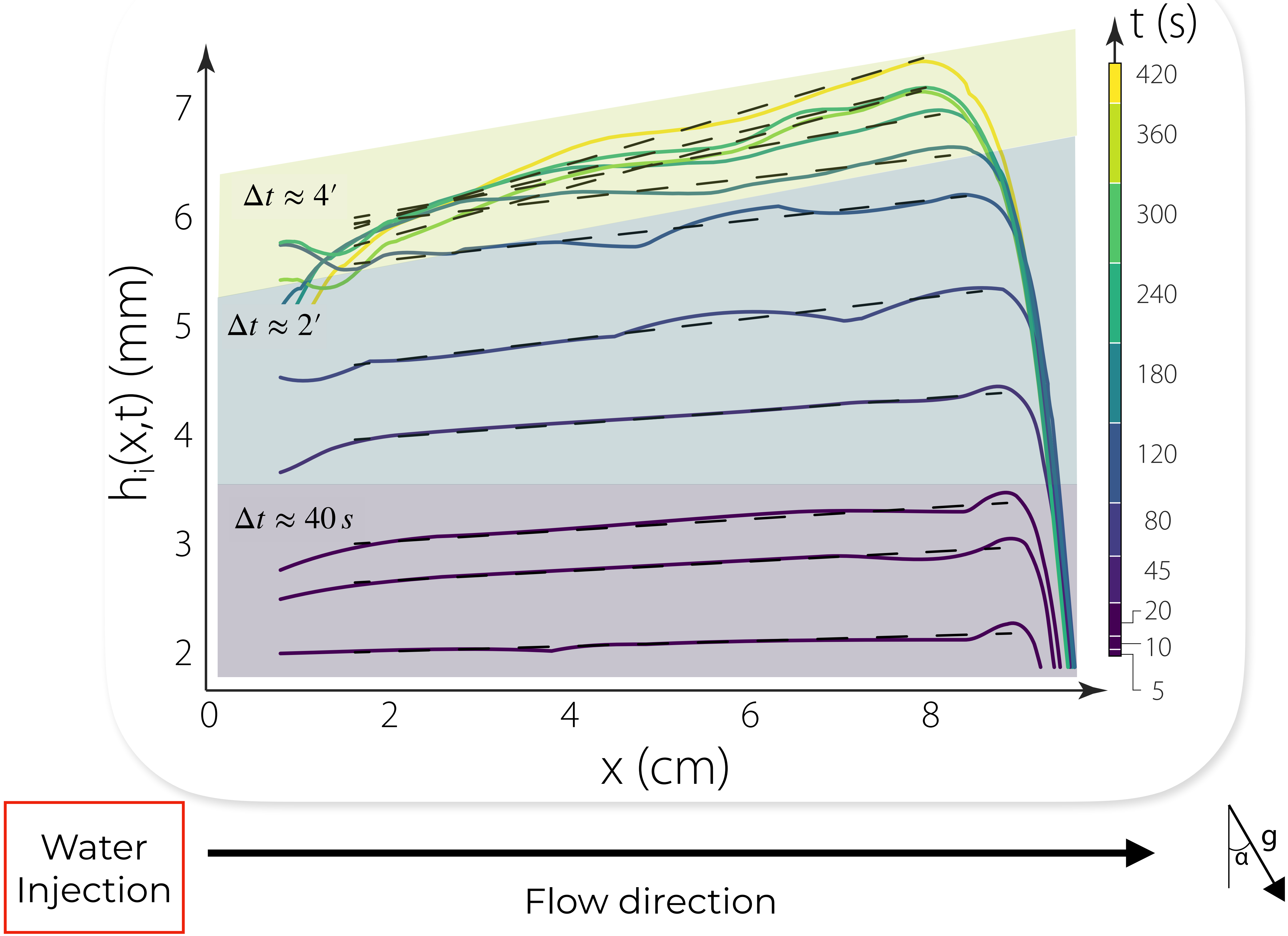
Injection temperature, T_{in}

Substrate temperature, T_s

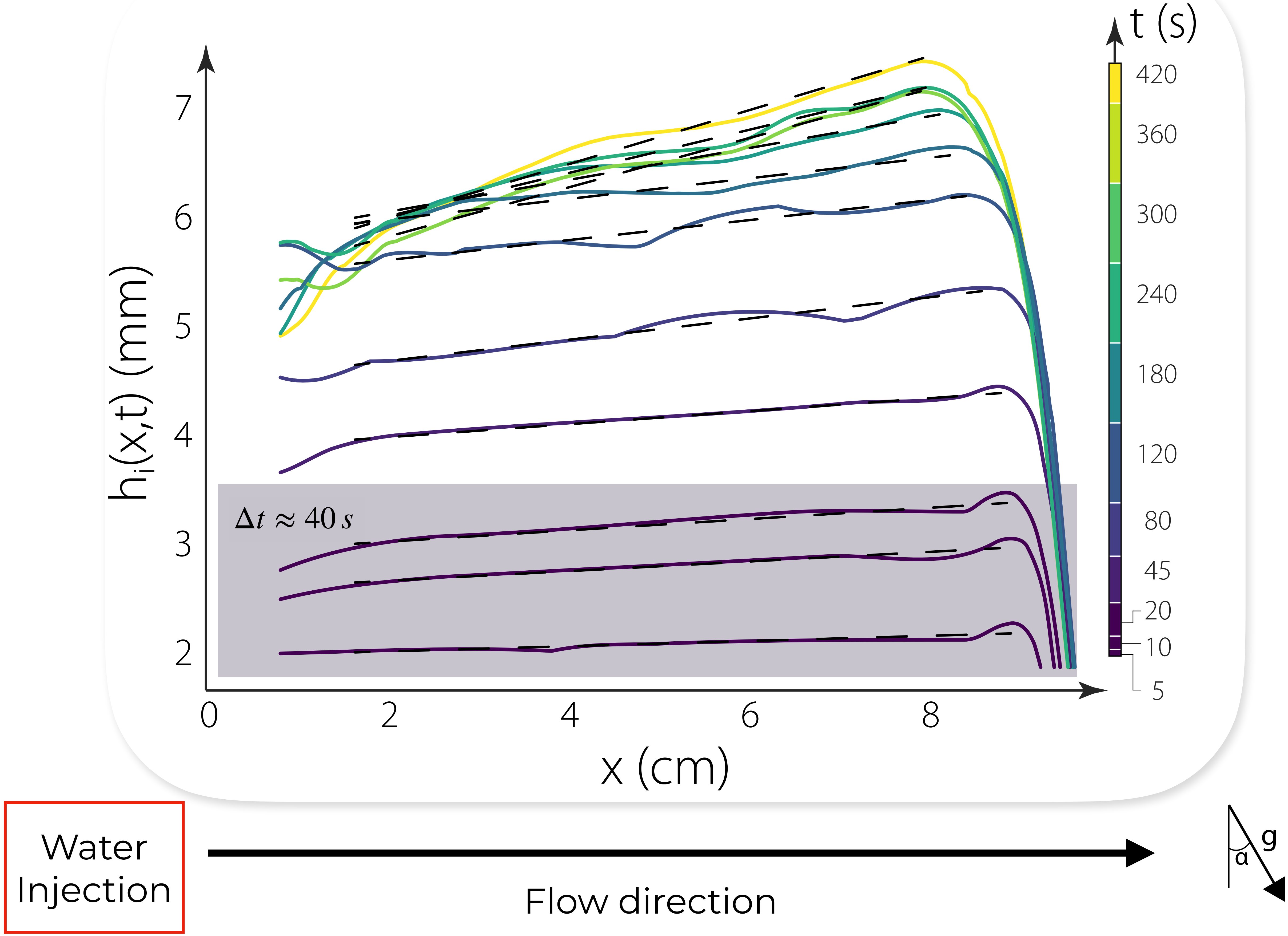
EXPERIMENTAL SETUP



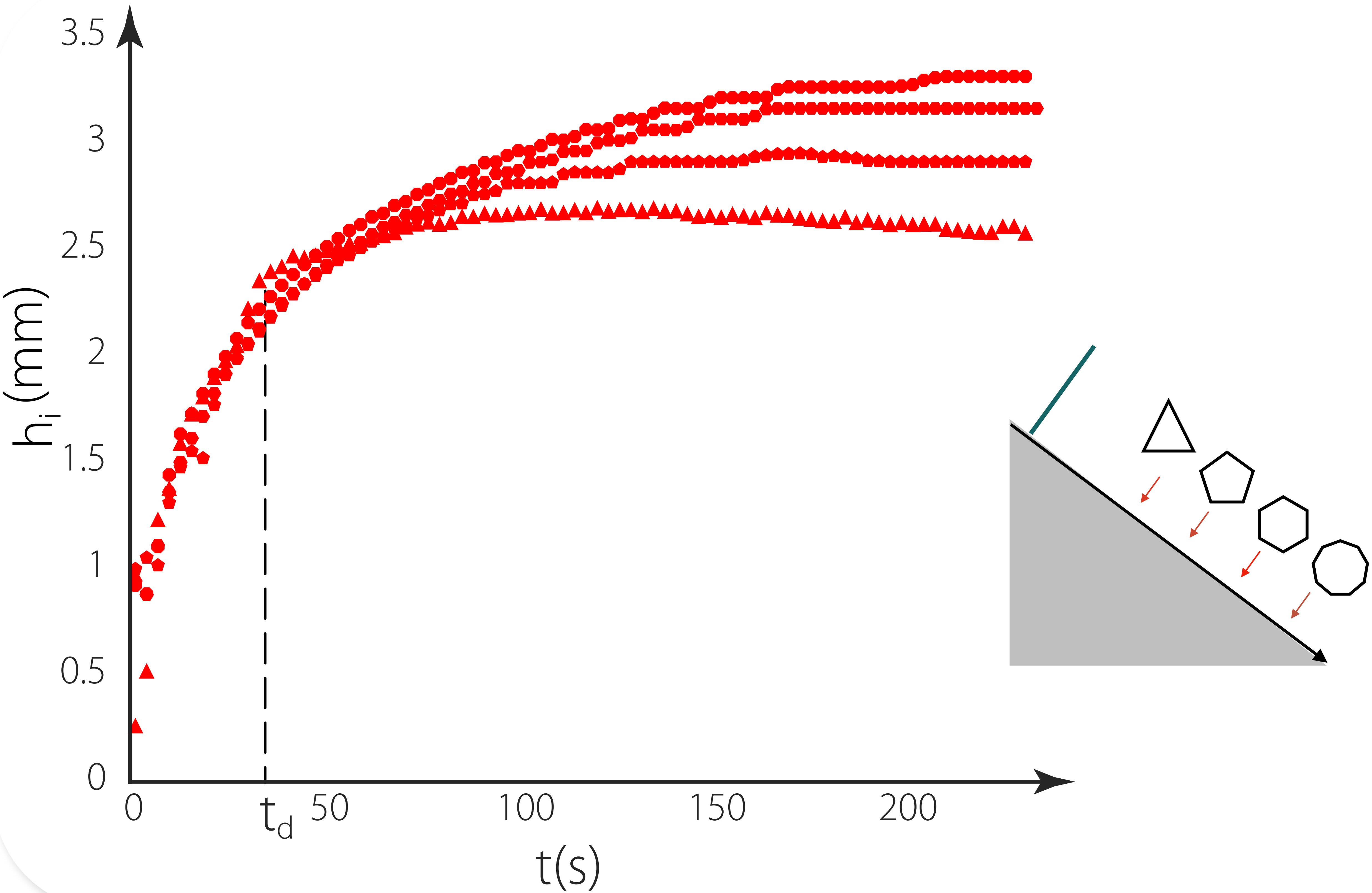
THREE REGIMES



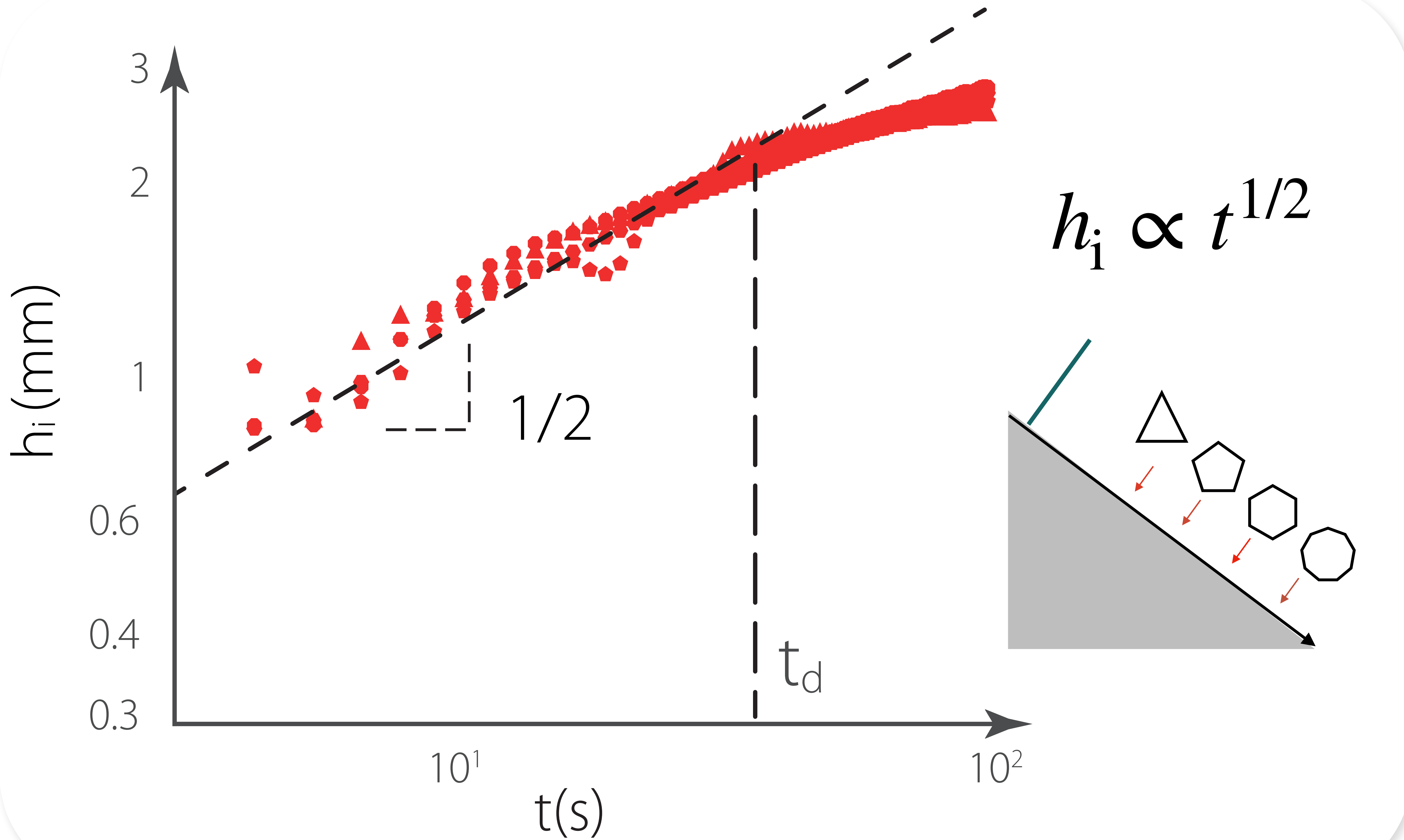
THREE REGIMES



INITIAL GROWTH: HOMOGENEOUS



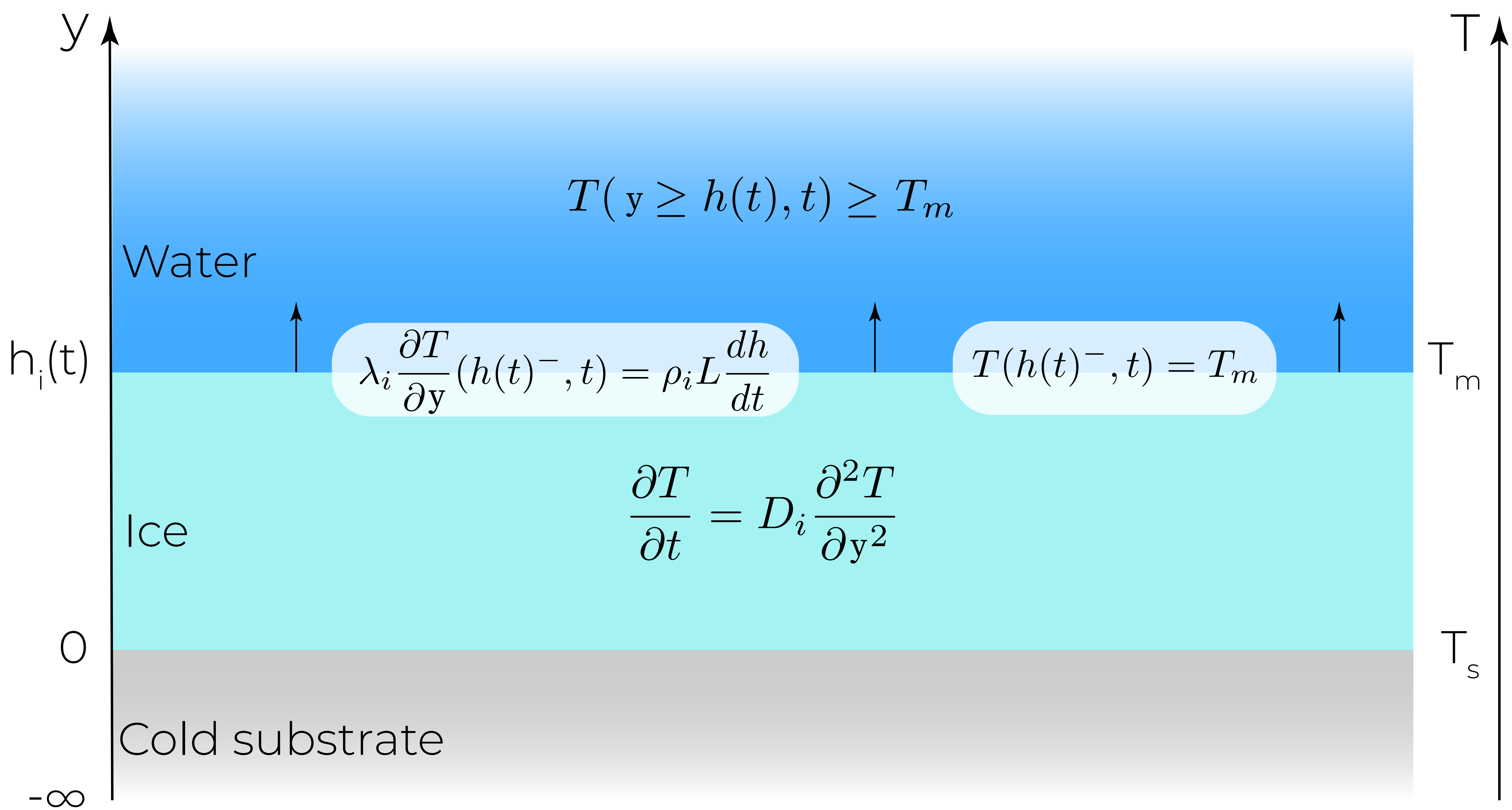
INITIAL GROWTH: DIFFUSIVE PROCESS



1D DIFFUSIVE PROCESS, STEFAN MODEL

Stefan Problem

1D, no flow, heat transfer model



INITIAL GROWTH: DIFFUSIVE PROCESS

Scaling law argument

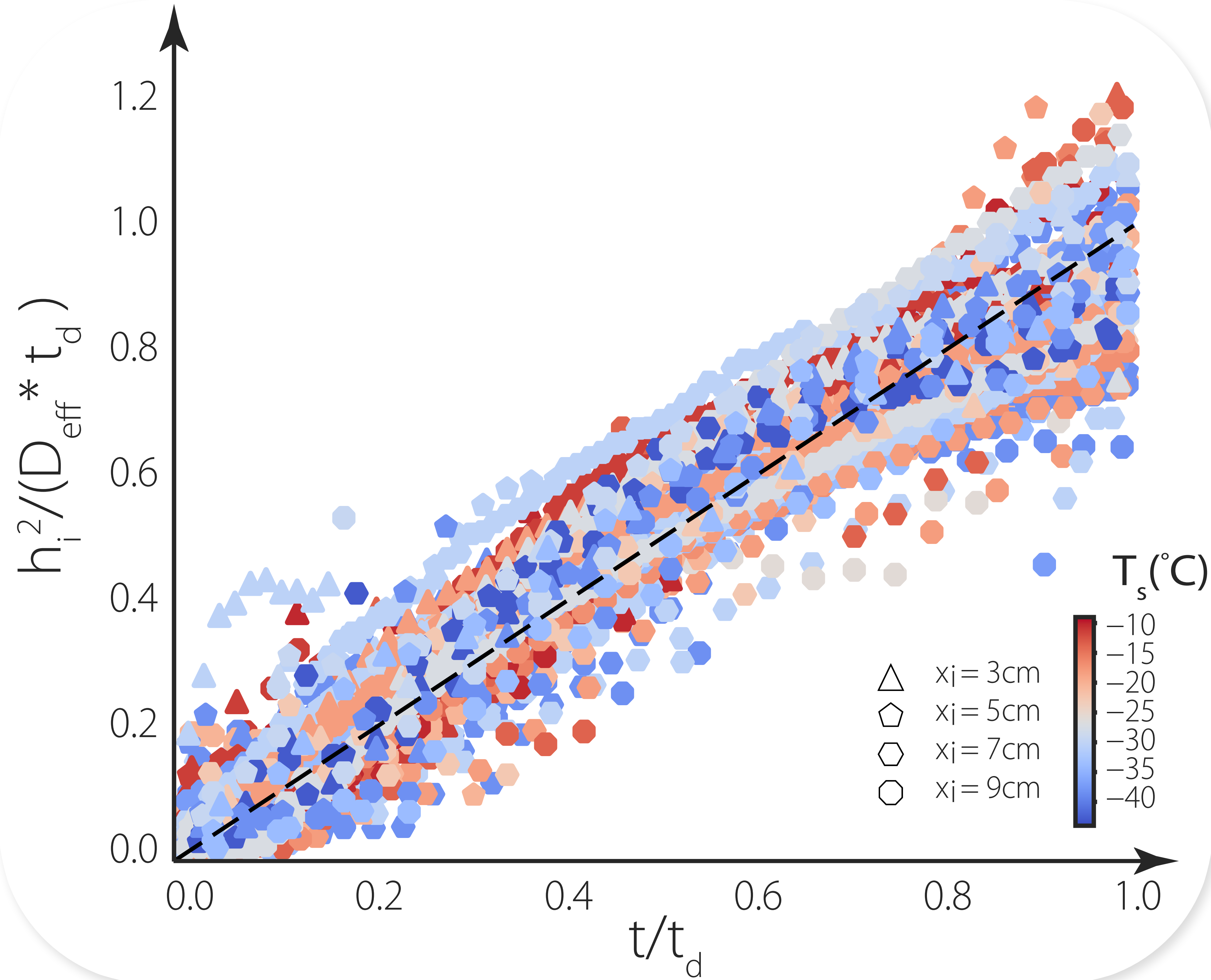
$$\rho_i L \frac{\partial h_i}{\partial t} = \lambda_i \frac{\partial T}{\partial z}$$

$$\rho_i L \frac{\partial h_i}{\partial t} \sim \lambda_i \frac{\Delta T}{h_i}$$

$$\Delta T = T_m - T_s$$

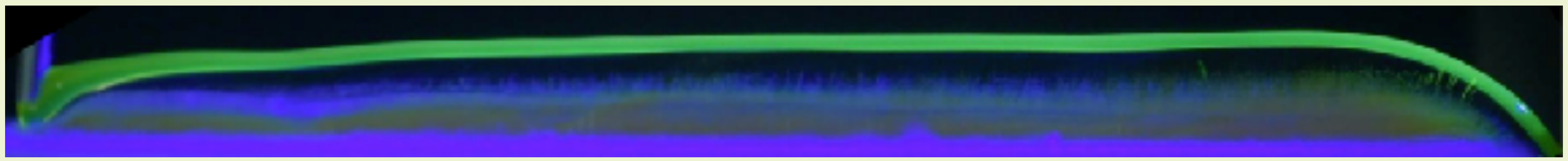
$$h_i^2 \sim \frac{\lambda_i \Delta T}{\rho_i L} t$$

D_{eff}

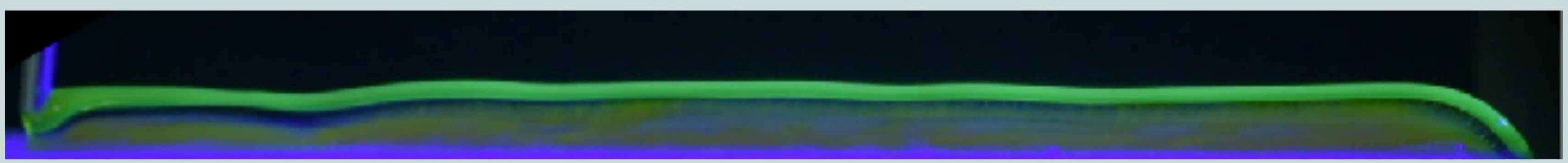


The growth process is similar to static conditions

BOUNDARY CONDITION REGIMES



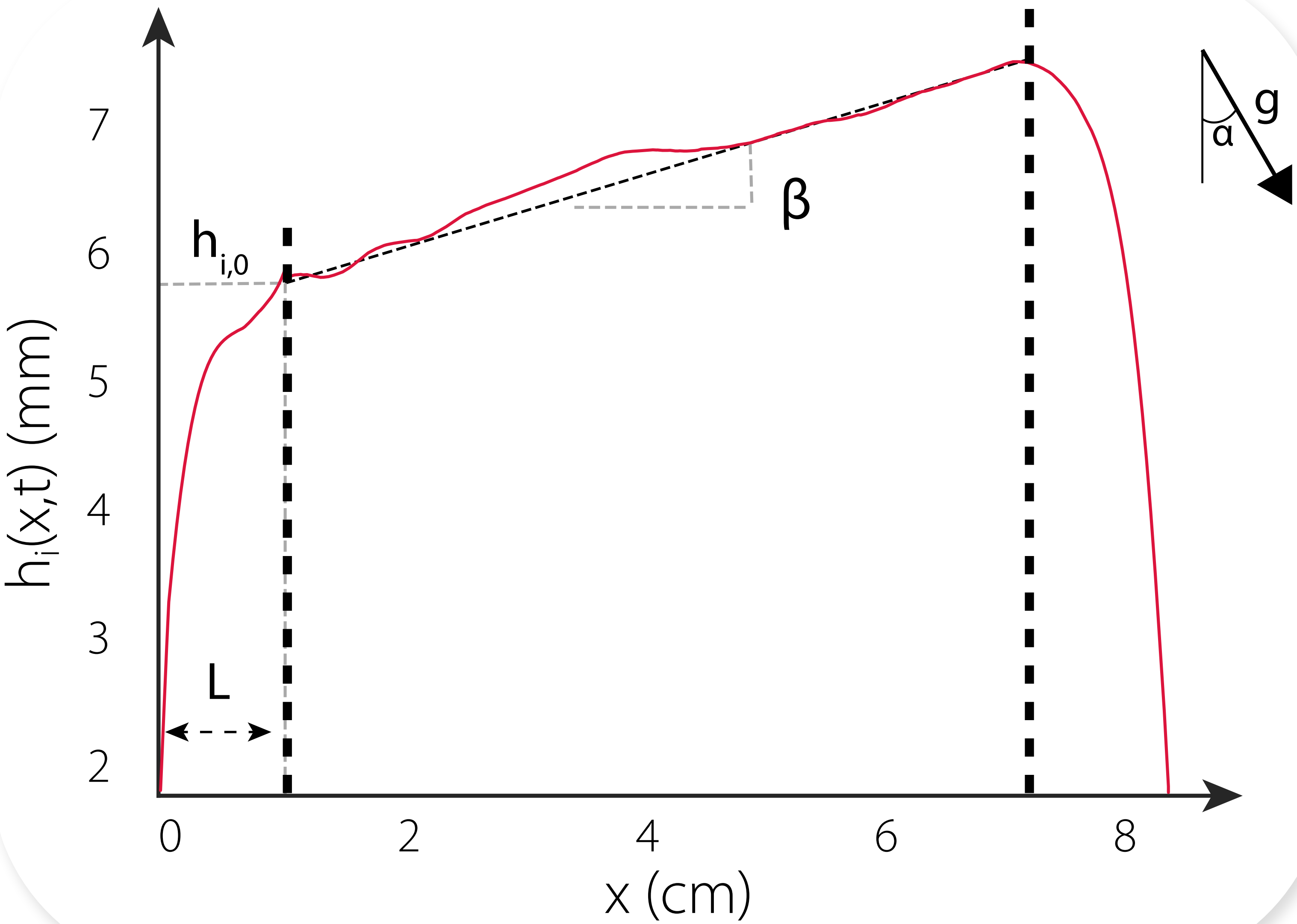
$$\cancel{\rho L \frac{\partial h}{\partial t}} = \lambda_{\mathrm{i}} \frac{\partial T}{\partial z} - \lambda_{\mathrm{w}} \frac{\partial T}{\partial z}$$



$$\rho L \frac{\partial h}{\partial t} = \lambda_{\mathrm{i}} \frac{\partial T}{\partial z} - \lambda_{\mathrm{w}} \frac{\partial T}{\partial z}$$

$$\rho L \frac{\partial h}{\partial t} = \lambda_{\mathrm{i}} \frac{\partial T}{\partial z} - \cancel{\lambda_{\mathrm{w}} \frac{\partial T}{\partial z}}$$

STATIC SHAPE OF THE ICE LAYER



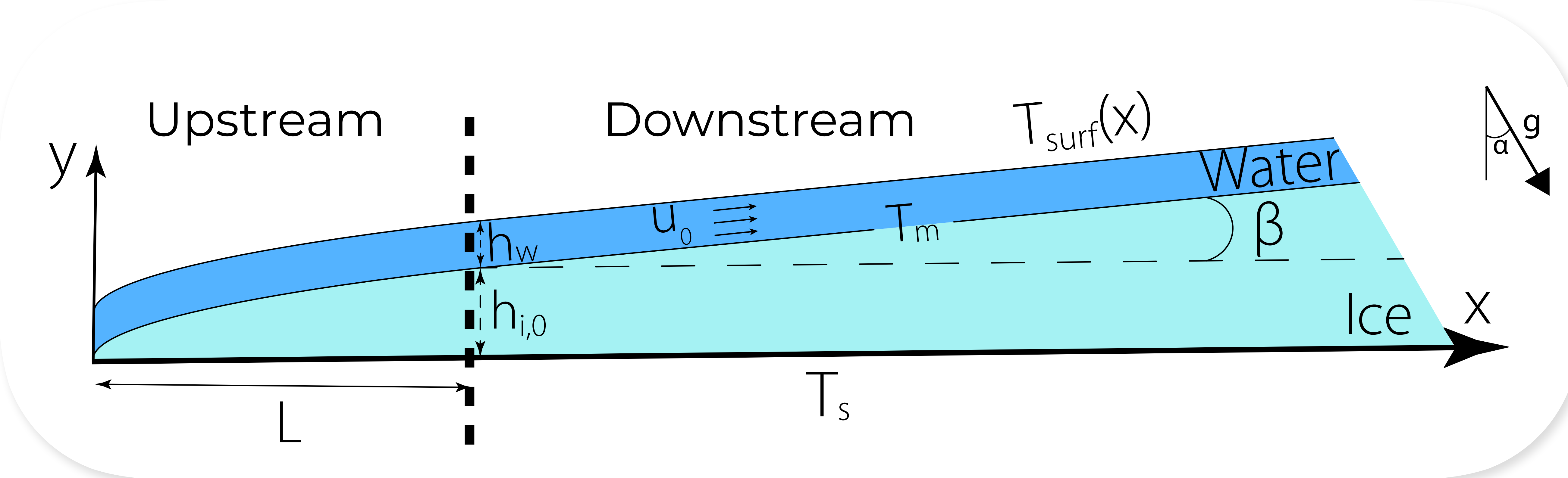
β	Ice slope
L	Entry length
$h_{i,0}$	Ice entry thickness

Ice thickness

$$h_i(x) = h_{i,0} + \beta x$$

- Recover the two regions and the shape of the ice at steady state ?
- Get a prediction for β ?

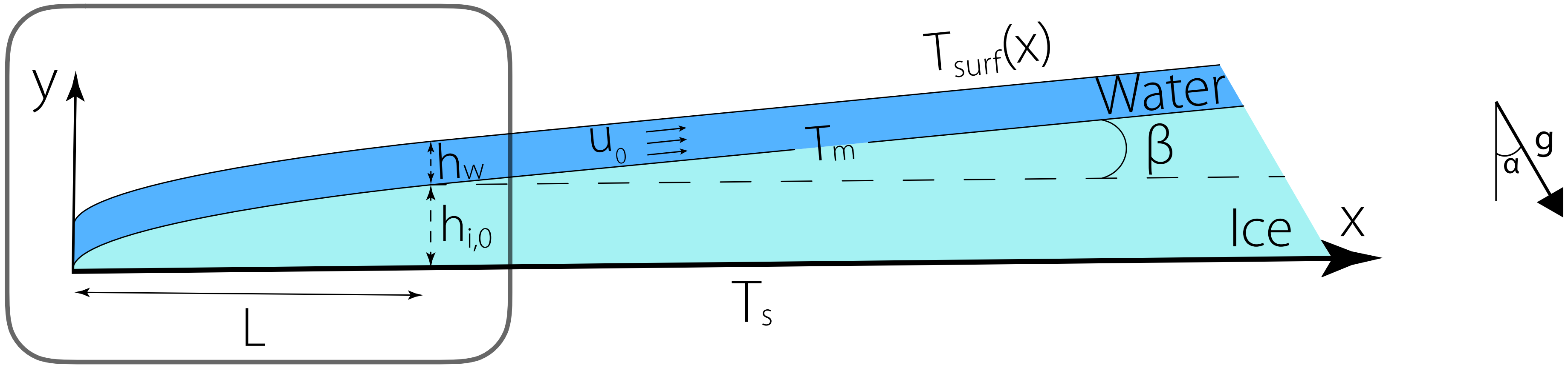
DESCRIPTION OF THE STATIC SHAPE

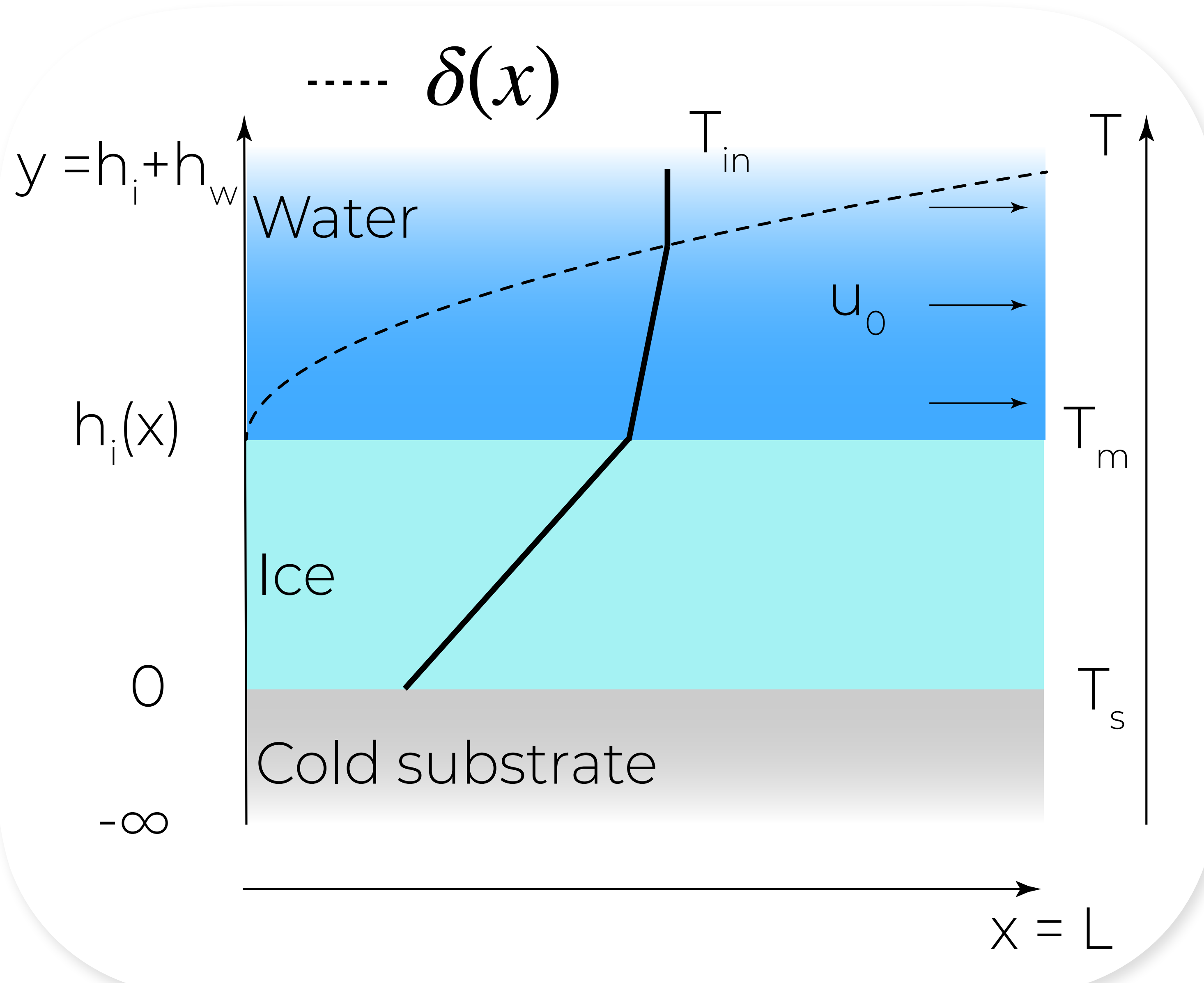


u_0	Mean flow velocity
h_w	Water thickness
T_{surf}	Water surface temperature

Upstream
vs
Downstream
heat transfer

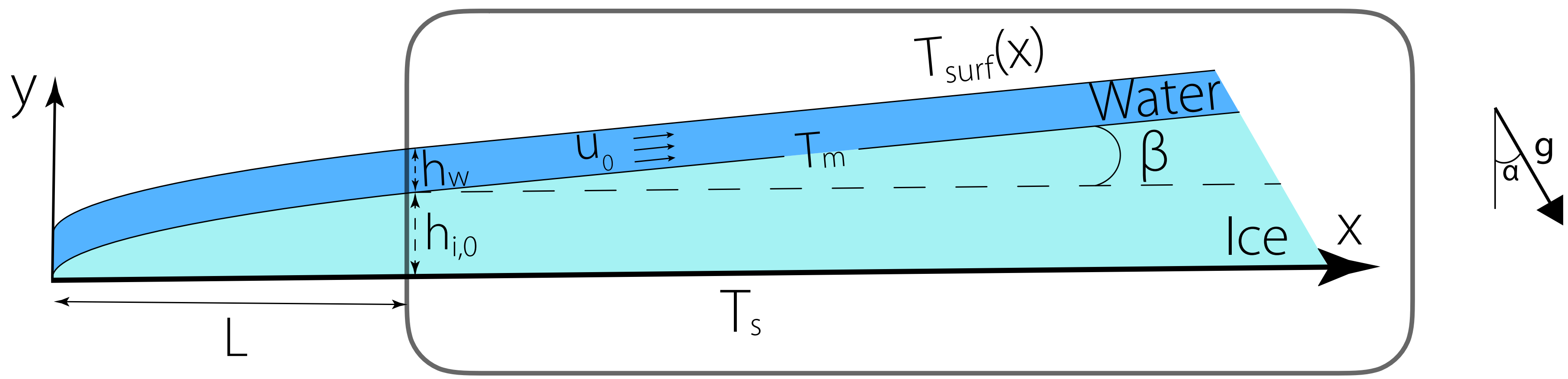
UPSTREAM HEAT TRANSFER

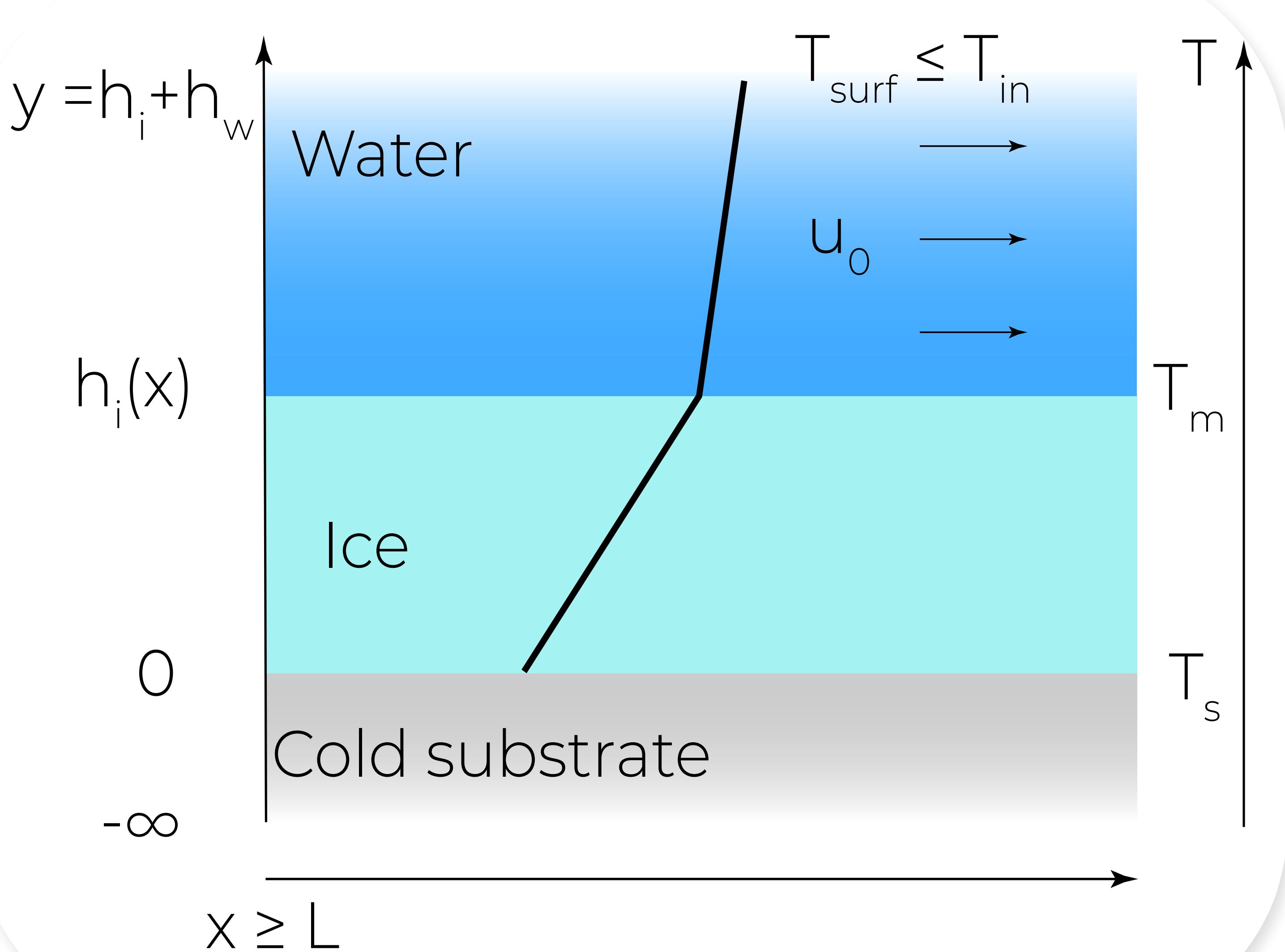




- Growth of a thermal boundary layer
- Water surface temperature remains T_{in}

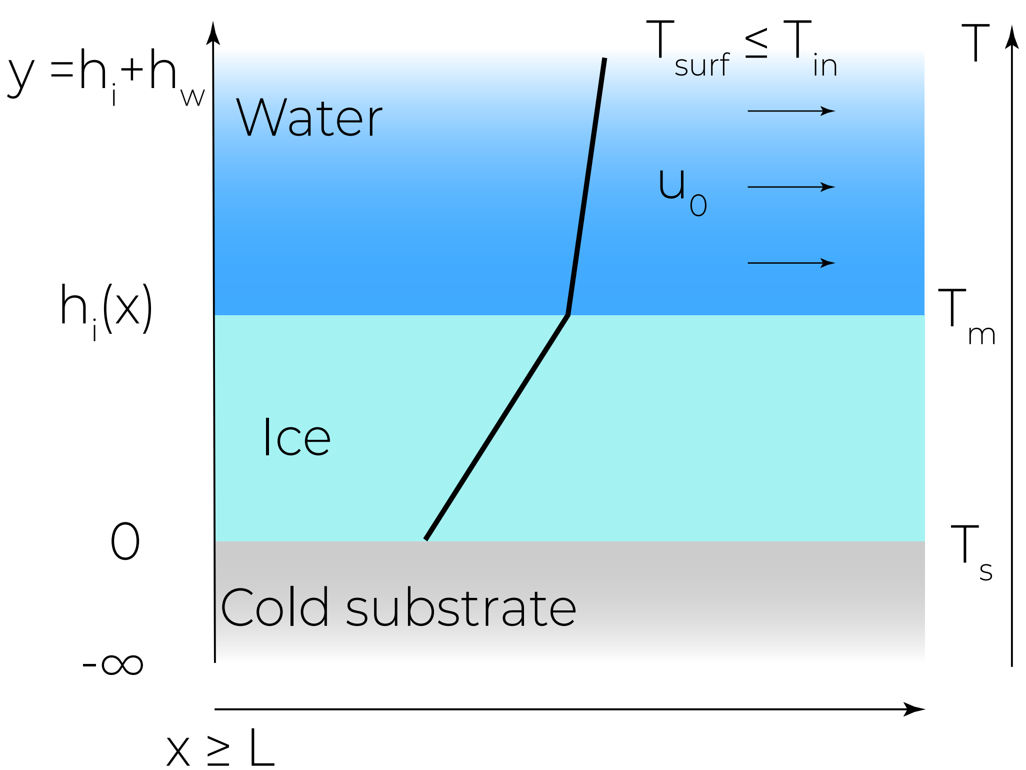
DOWNSTREAM HEAT TRANSFER





- Confinement of the thermal boundary layer
- Water surface temperatures lowers with plane position

SURFACE TEMPERATURE FIELD



Advection-diffusion in water

$$u_0\, \partial_x T = D_w \partial_{yy} T$$

- No heat transfer towards the air
- Melting temperature at the ice/water interface

Heat flux balance

$$\lambda_i \frac{T_m - T_s}{h_i(x)} \sim \lambda_w \frac{T_{surf}(x) - T_m}{h_w}$$

$$h_i(x) \sim \frac{\lambda_i\, h_w}{\lambda_w} \frac{T_m - T_s}{\boxed{T_{surf}(x)} - T_m}$$

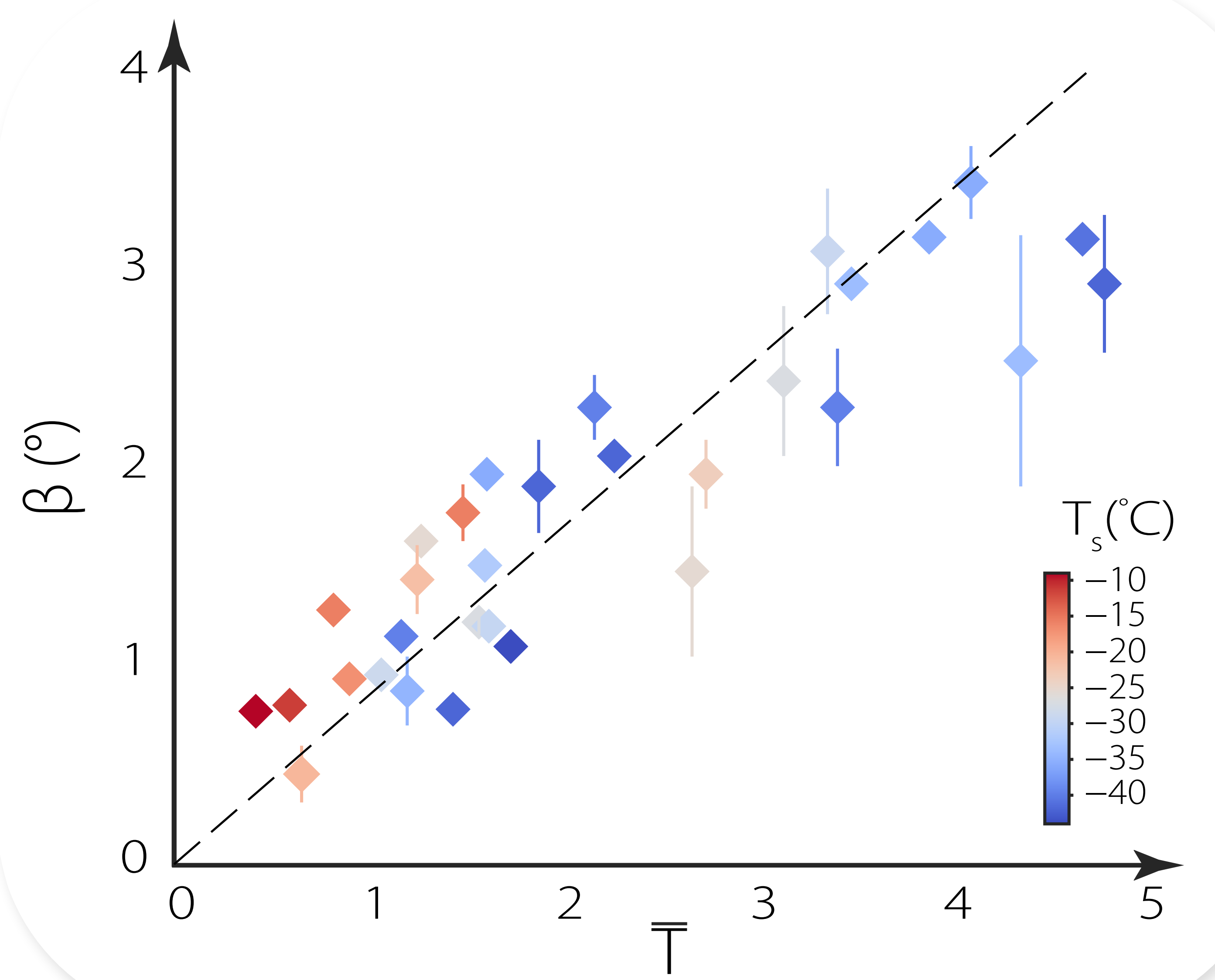
Simplified Graetz Problem

$$T_{surf}\,(x) \sim T_{in} - a\, \left(T_{in} - T_m\right) \frac{x - L}{L}$$

VARIATION OF THE SLOPE β

$$\begin{cases} h_i(x) \sim h_w \dfrac{\lambda_i}{\lambda_w} \dfrac{T_m - T_s}{T_{in} - T_m}\left(1 + a\dfrac{x-L}{L}\right) & \text{Model} \\ h_i(x) = h_{i,0} + \beta x & \text{Experimental} \end{cases}$$

$$\beta \propto \frac{T_m - T_s}{T_{in} - T_m} = \overline{T}$$



30 experiments

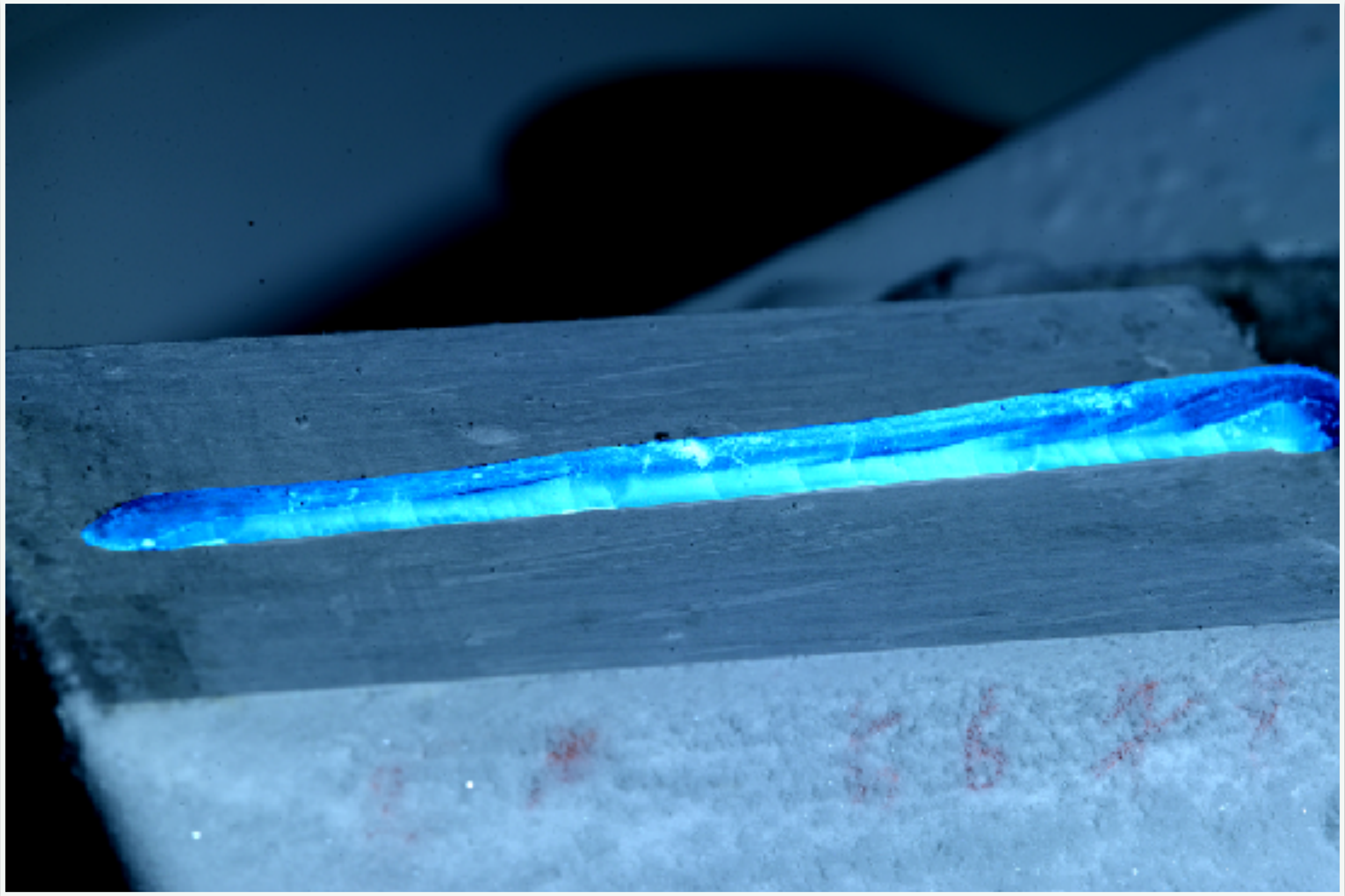
(T_s, T_{in})

CONCLUSIONS

- Singular structure formed by a frozen rivulet

- Quasi-static initial growth

- Confinement of a thermal boundary layer



- Coupling hydrodynamics and solidification

Monier et al., ArXiv, 2019