EFDC1

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A **THERMAL WALL LAW** FOR HEAT TRANSFER PREDICTION IN **LAMINAR FLOW** AT **HIGH PRANDTL NUMBER**: APPLICATION TO ELECTRICAL MACHINES

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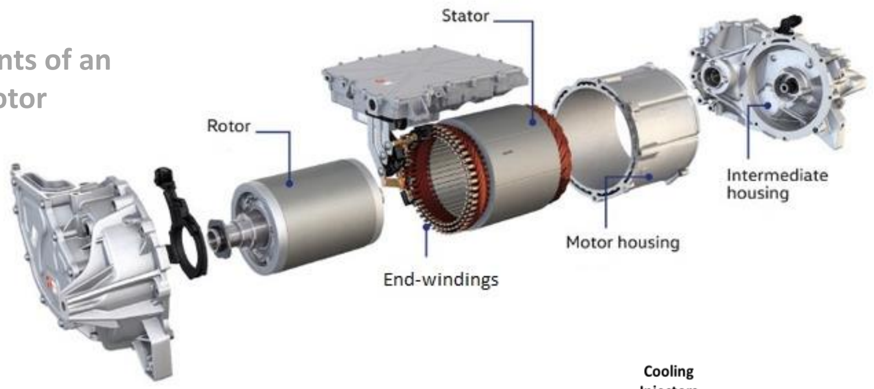
INDUSTRIAL CONTEXT

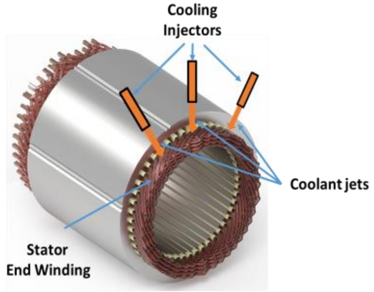
- Industrial context of **electrical motors** for **electric vehicles**

Main components of an electric motor



- Local cooling of the **end-windings** through **oil jets**

Direct liquid cooling of the stator end-windings



- Physics of the oil jets cooling :

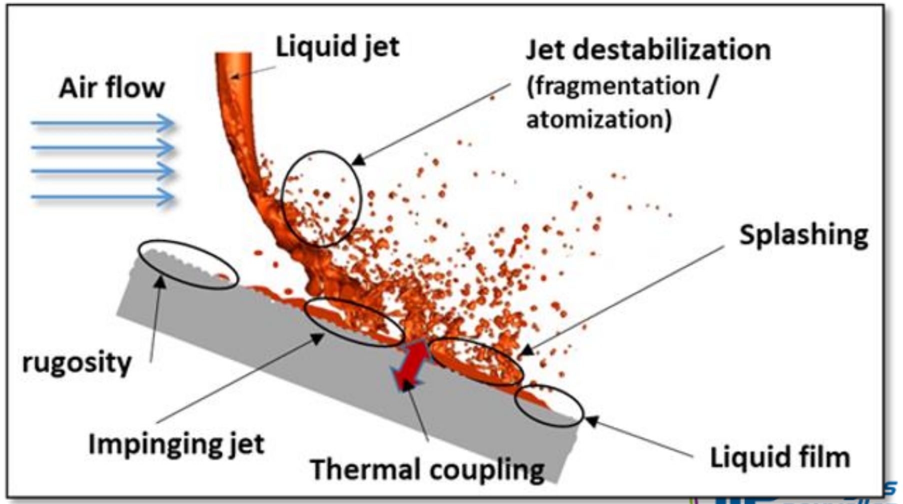
  Liquid film formation

  Laminar flows *[1]* $(Re_{jet} < 1000)$

  Complex phenomena: *[2,3]*

  - Cross-air flow, Splashing
  - Liquid film formation with various dynamic behavior, liquid film instabilities
  - Surface wetting prediction, complex surfaces
  - **High Prandtl number liquid cooling**

Complex phenomena involved

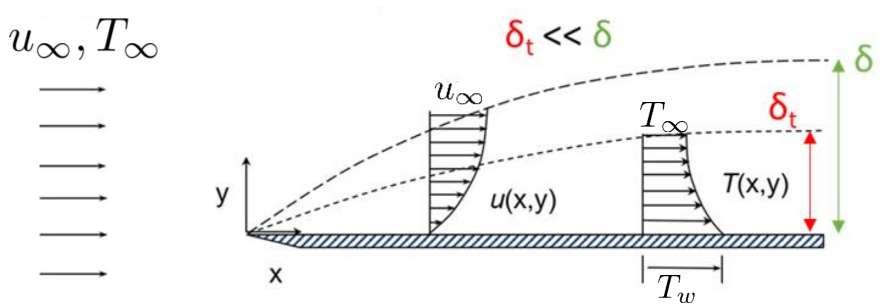


[1] Poubeau et al. 2024 [2] Sindjui et al. 2022; [3] Poubeau et al. 2023;

CONTEXT & OBJECTIVE OF THE PRESENT WORK

- Specifically addressed in the present work : **numerical simulation of high Prandtl number liquid cooling**
  - $100 < Pr < 400$
  - Thin thermal boundary layer
  - Multi-physical scales in one simulation: **high computational cost**

Boundary layers illustration (Pr >> 1)



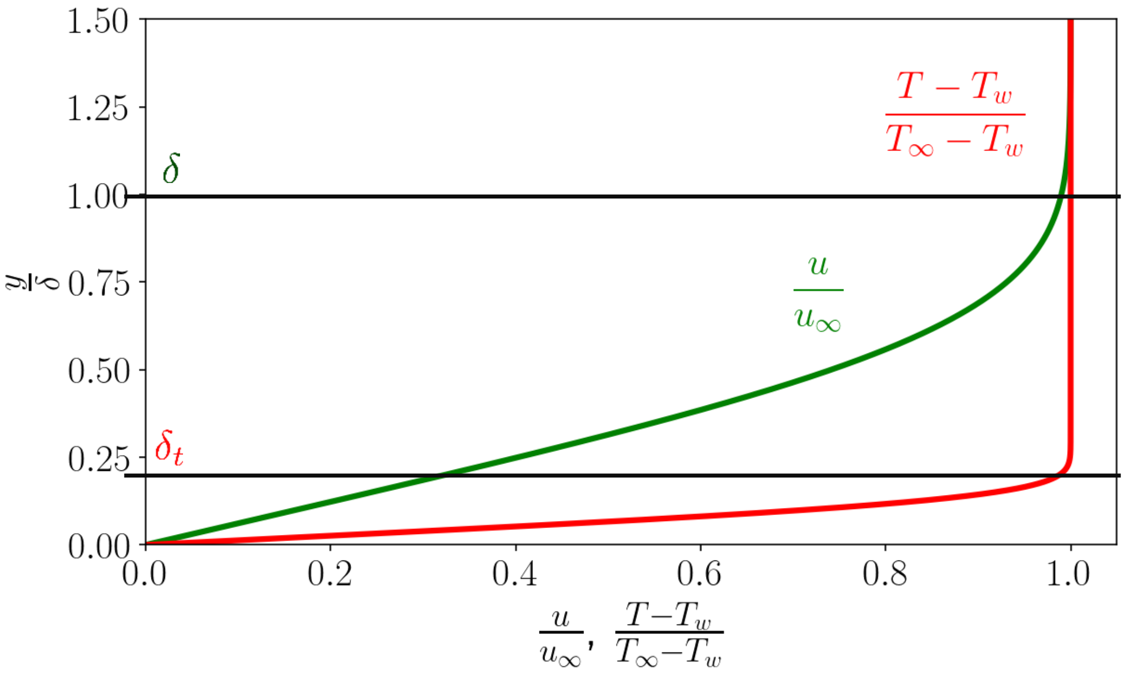
- Objective:

  Propose a **thermal wall law** that eliminate the need to finely mesh the thermal boundary layer and thus reduce computational cost.

HIGH PRANDTL NUMBER FLUID FLOW CHARACTERISTIC

- Prandtl number definition: $Pr = \frac{\nu}{\alpha}$
  - ν : Momentum diffusivity
  - α : Thermal diffusivity
- P_r >> 1 :
  - $\delta >> \delta_t$
  - Velocity in the **linear part of the velocity profile**
- $P_r \rightarrow \infty$:
  - Temperature field **solved by Lévêque analytically** *[2]*

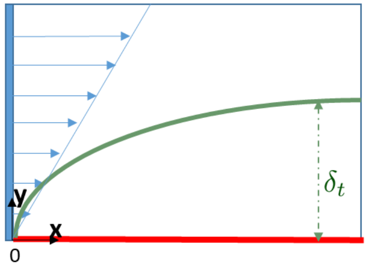
Boundary layer development on a flat plate (Pr = 100): Scaled velocity and Temperature profile solved by Blasius *[1]*



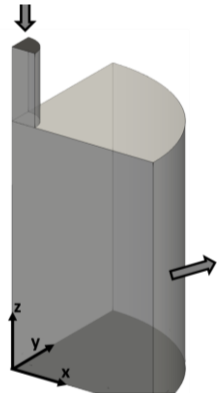
[1] Blasius 1908; [2] Lévêque 1928

SUMMARY OF THE PRESENTED WORK

- 1: Propose a thermal wall law model based on Lévêque theory
- 2: Assess the law on single-phase flow verifying wall law hypothesis



- 3: Assess the law on academic impinging jet configuration



Using a **finite volume approach** via CONVERGE CFD SOFTWARE (v 3.1.4)

1 – THERMAL WALL LAW DESCRIPTION

THERMAL WALL LAW DESCRIPTION

Description of the model:

$$q_w = f(T_1, T_w, T_\infty, y_1, k_1)$$

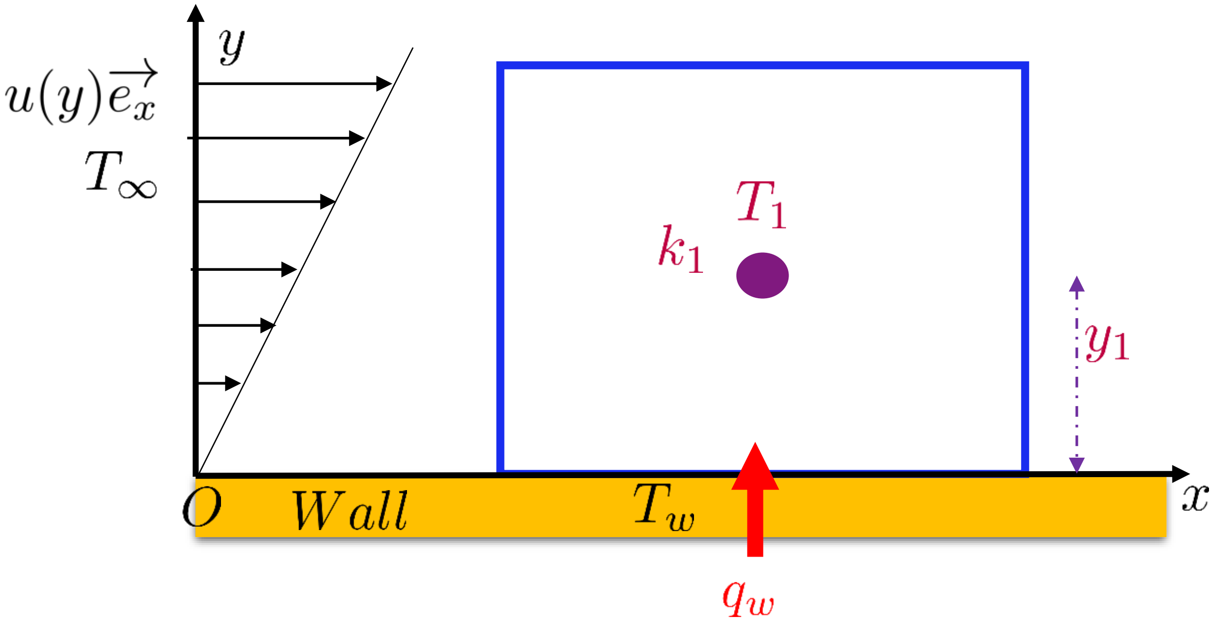
Hypothesis behind the model:

The common b.l. hypothesis:

- Stationary, 2D, incompressible flow
- Flat wall
- Uniform properties and uni. mean velocity outside the b.l.
- Low viscous dissipation
- $\frac{\partial}{\partial y} >> \frac{\partial}{\partial x}$

Specific hypothesis for the Lévêque theory:

- T_w = uniform
- v = 0 (velocity y-component null)
- u(y) ~ y, uniform slope along the wall



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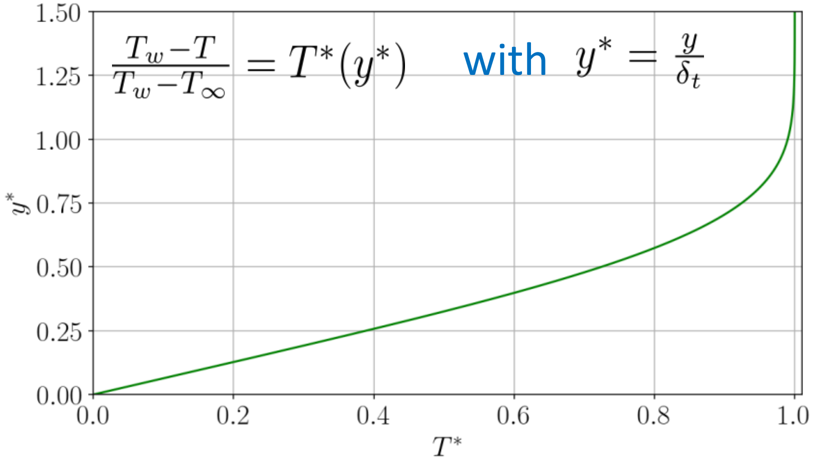
Model based on **Lévêque theory**:

- Lévêque solves energy equation:

$$u\frac{dT}{dx} = \alpha\frac{d^2T}{dy^2} \quad \text{with} \quad \begin{cases} T(y=0) = T_w \\ T(y\to\infty) = T_\infty \\ \frac{\partial T}{\partial y}(y\to\infty) = 0 \end{cases}$$

- T° profile remain self-similar (consistent with experimentations):



- Solution for q_w and δ_t:

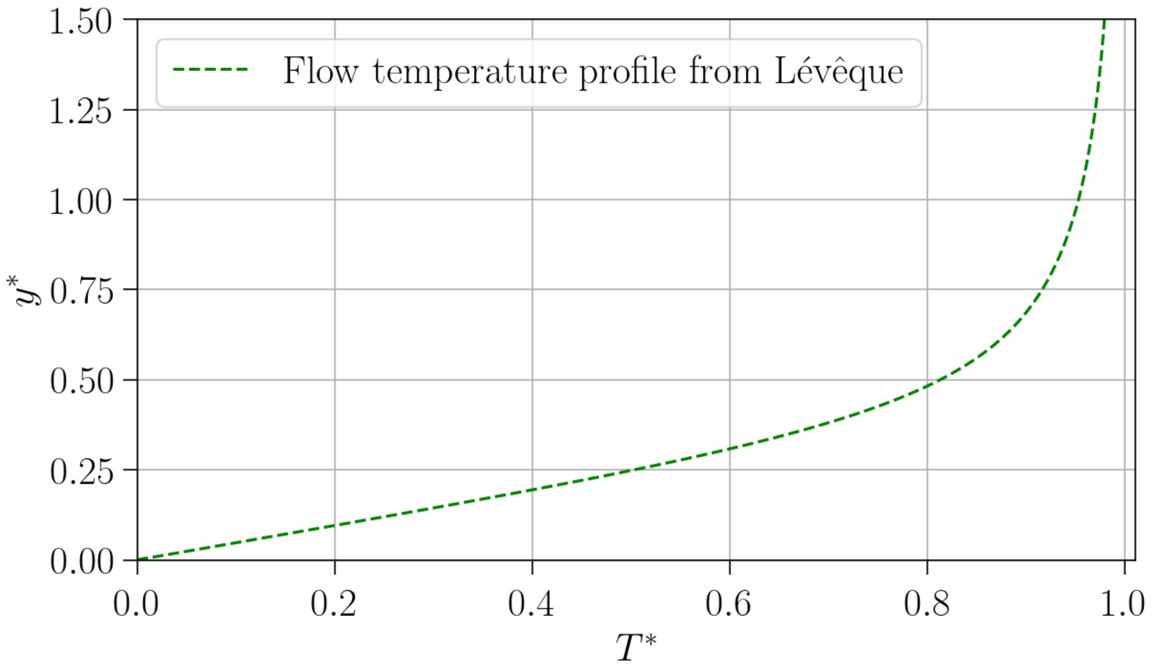
$$\delta_t \approx 2.95 \times \left(\frac{x\ \alpha}{\left.\frac{\partial u}{\partial y}\right|_0}\right)^{\frac{1}{3}} \quad \text{with} \quad \alpha = \frac{k}{\rho\ C_p}$$

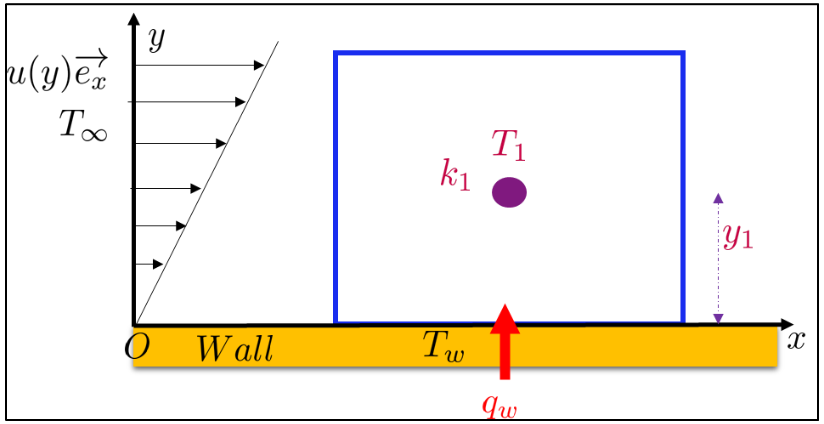
$$q_w \approx 1.593\ k(T_w - T_\infty)\delta_t^{-1}$$

- Thermal wall law using near-wall cell information:

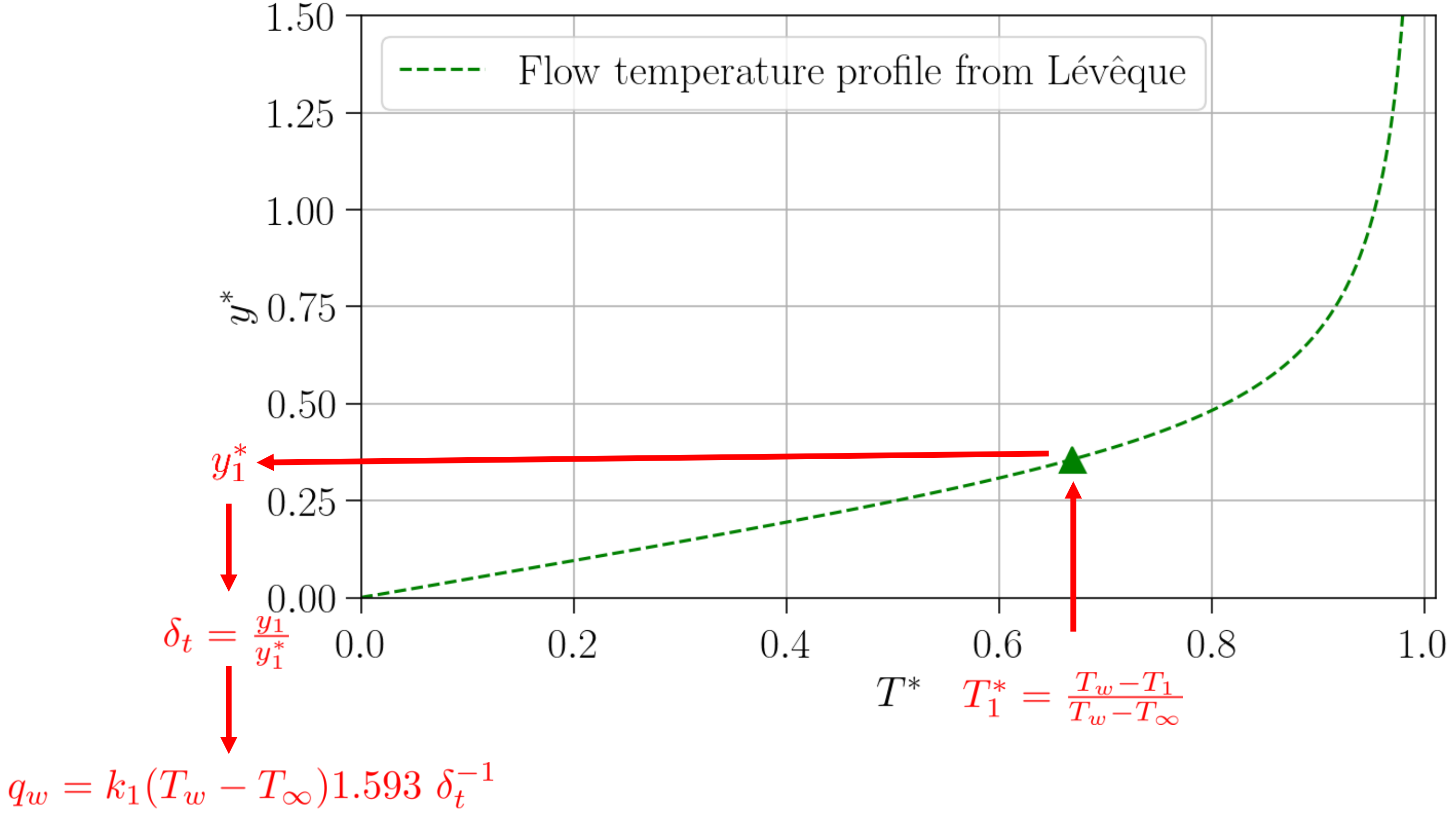
$$q_w = f(T_1, T_w, T_\infty, y_1, k_1)$$

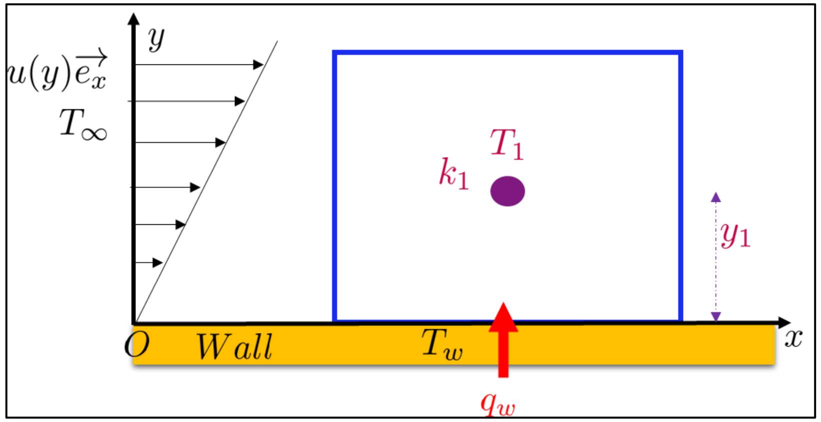
- Thermal wall law using analytical results from **Lévêque theory**:
  1. self-similar local temperature profile $T^*(y^*)$ with $y^* = \frac{y}{\delta_t}$ and $T^* = \frac{T_w - T}{T_w - T_\infty}$
  2. Relation between q_w and δ_t : $q_w = k_1(T_w - T_\infty)1.593\ \delta_t^{-1}$





- Thermal wall law using analytical results from **Lévêque theory**:
  1. Autosimilar local temperature profile $T^*(y^*)$ with $y^* = \frac{y}{\delta_t}$ and $T^* = \frac{T_w - T}{T_w - T_\infty}$
  2. Relation between q_w and δ_t : $q_w = k_1(T_w - T_\infty)1.593\ \delta_t^{-1}$





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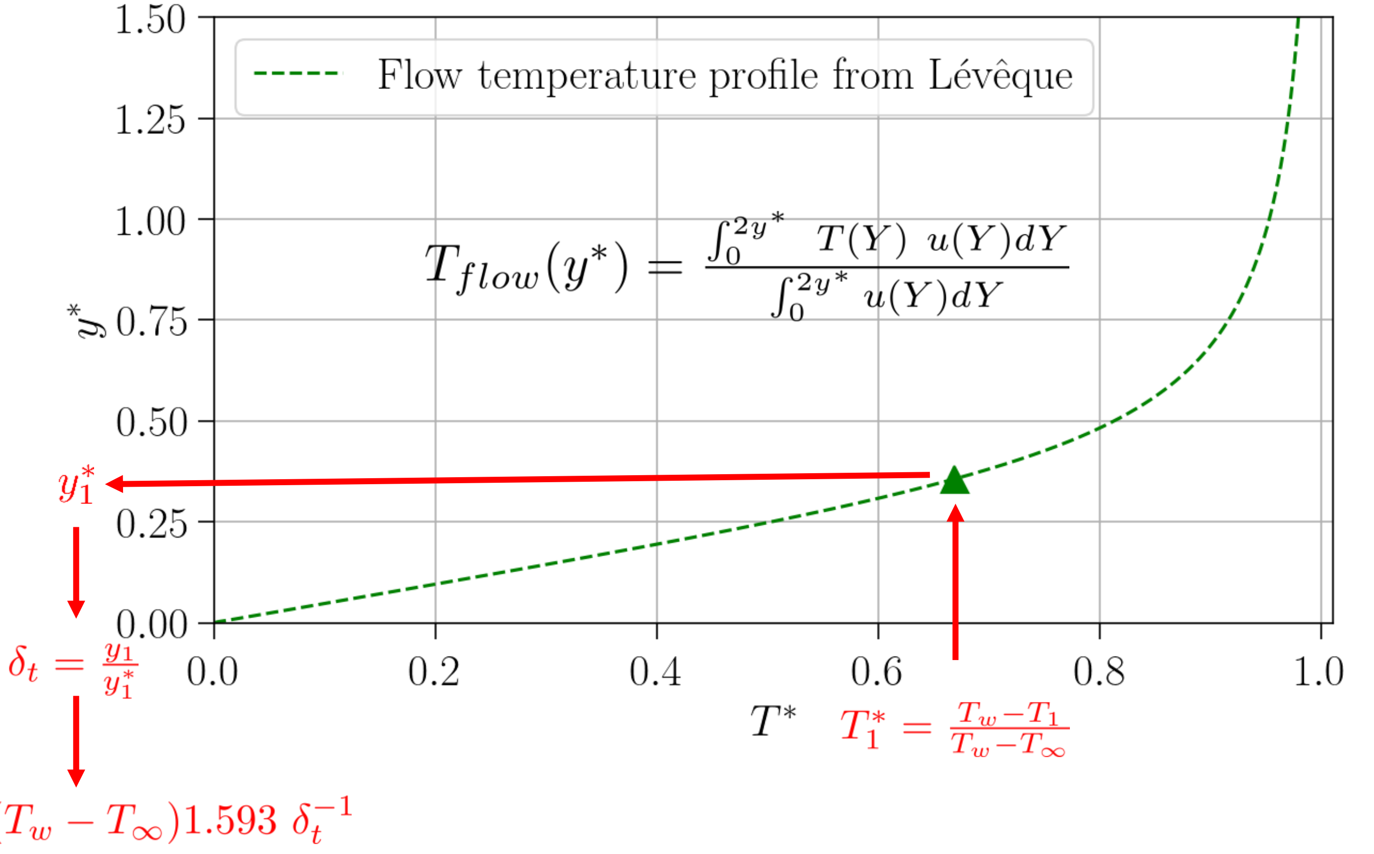

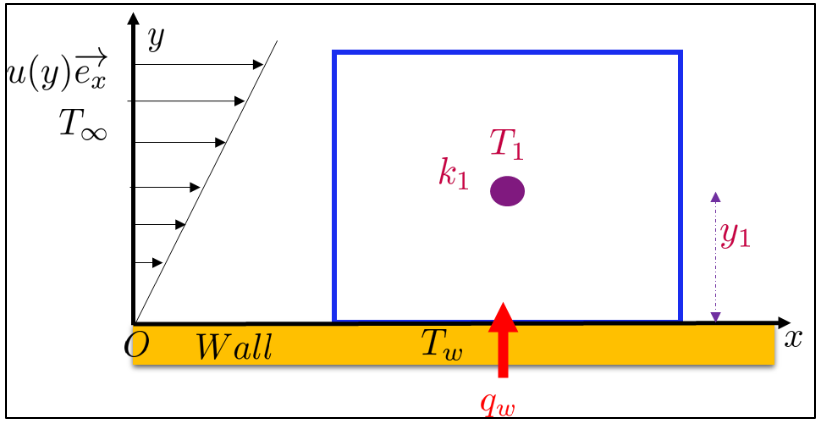

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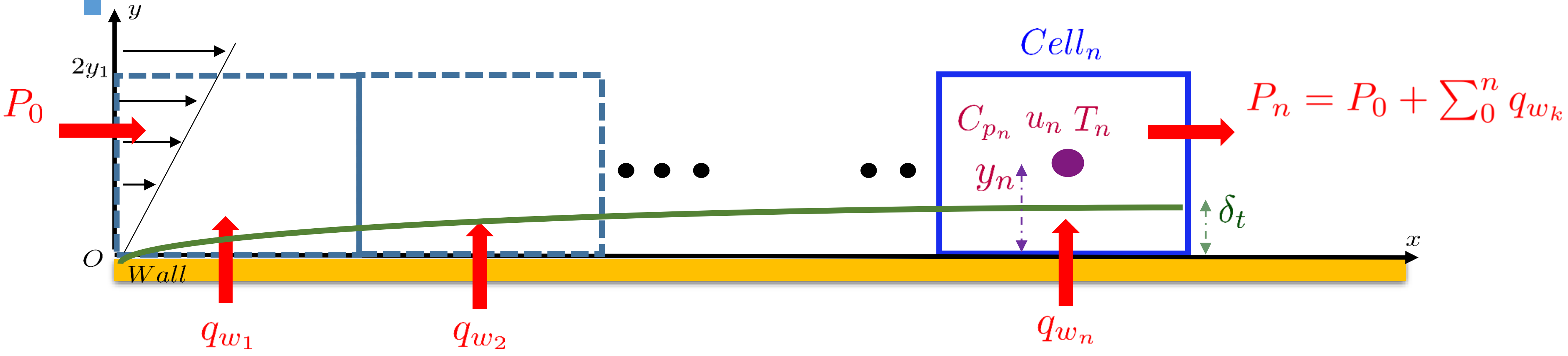


- In the finite volume approach, the **convective flux P_n** is expressed numerically in function of the cells' temperatures. For instance, with an upstream scheme we have:

$$P_n = 2y_n u_n \rho_n \, C_{p_n} T_n$$

- The exact analytical calculation is expressed as:

$$P_n = \int_0^{2y_n} \rho \, C_p \, T \, u(y) dy$$

- In the finite volume approach **T_n must be equal to a "flow" temperature to estimate P_n correctly:**

$$T_n = \frac{\int_0^{2y_n} \rho \, C_p \, T \, u(y) dy}{2y_n u_n \rho_n \, C_{p_n}}$$

if we suppose ρ, C_p uniform and:

$\rho_n = \rho$

$C_{p_n} = C_p$

Plus linear velocity profile: $u_n = \int_0^{2y_1} u(y) dy$

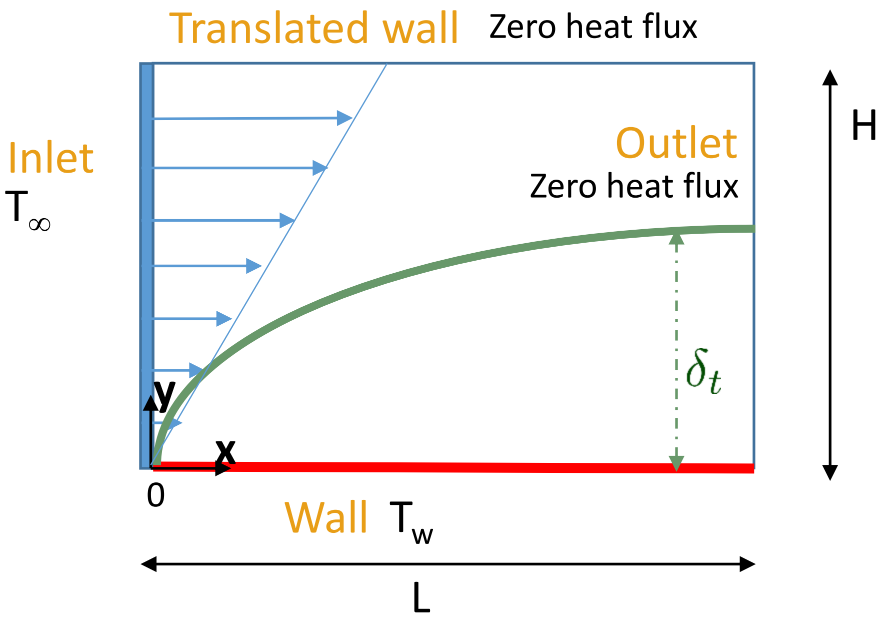
$$T_n = \frac{\int_0^{2y_n} T \, u(y) dy}{\int_0^{2y_n} u(y) dy}$$

ASSESS THERMAL WALL LAW

ON SINGLE-PHASE FLOW VERIFYING WALL LAW HYPOTHESIS

SINGLE-PHASE FLOW VERIFYING WALL LAW HYPOTHESIS

➢ Numerical set-up:



- Uniform cartesian mesh

SINGLE-PHASE FLOW VERIFYING WALL LAW HYPOTHESIS

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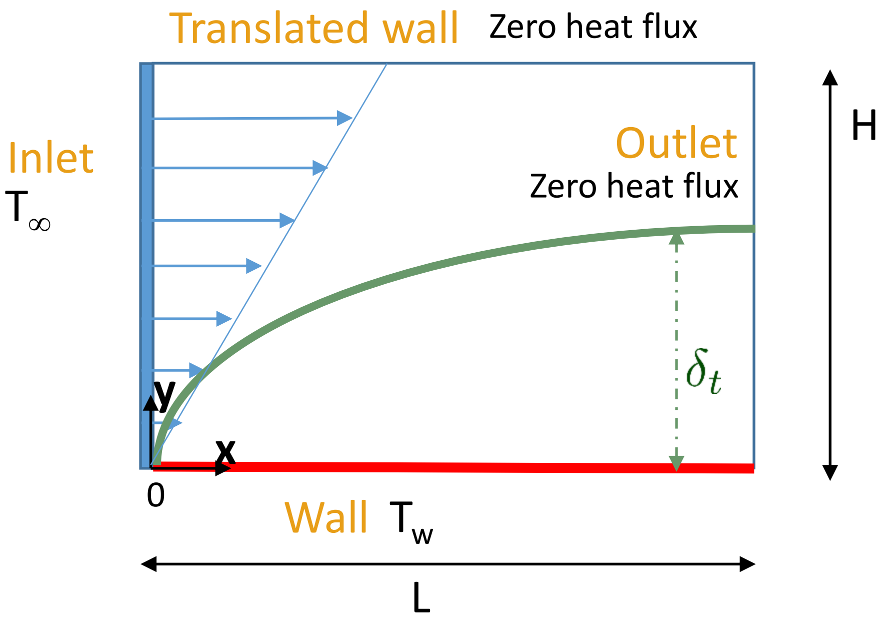

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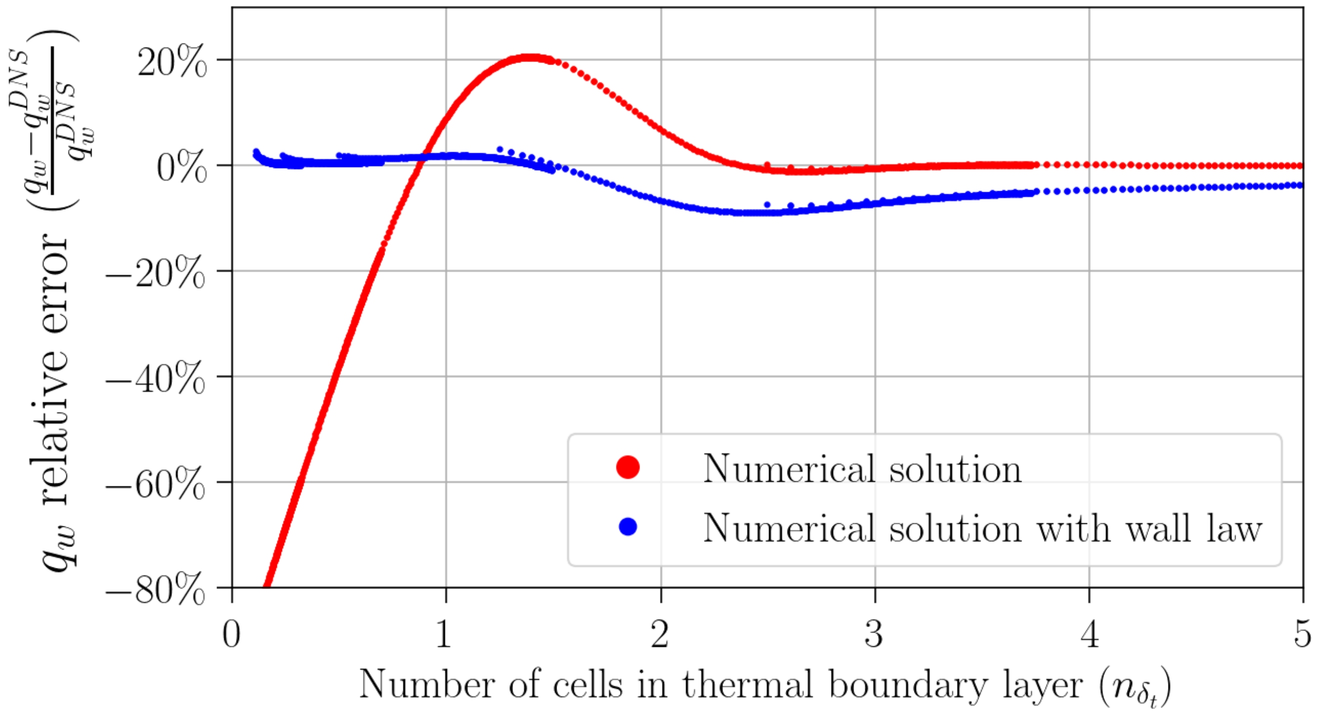
➤ Wall heat flux error with & without thermal wall law:



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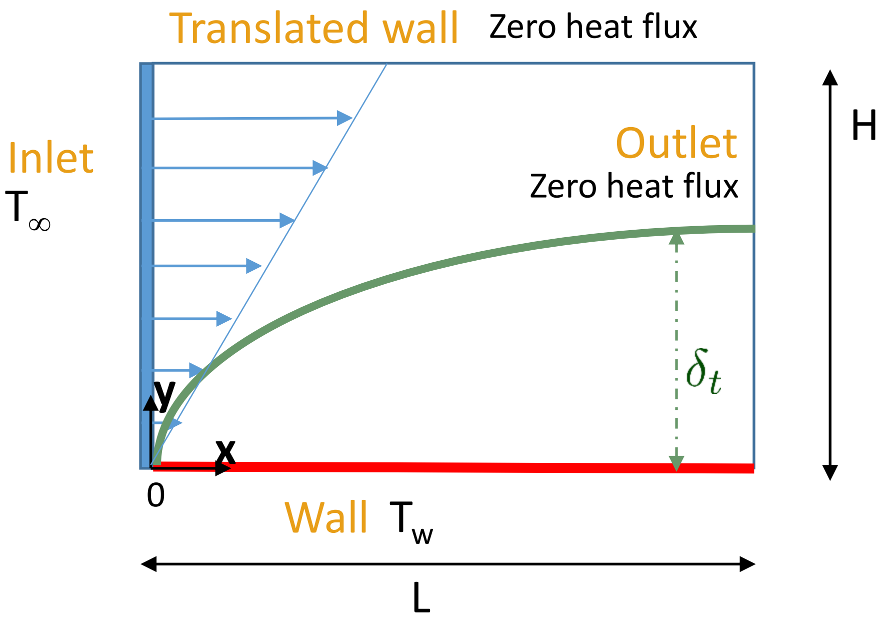

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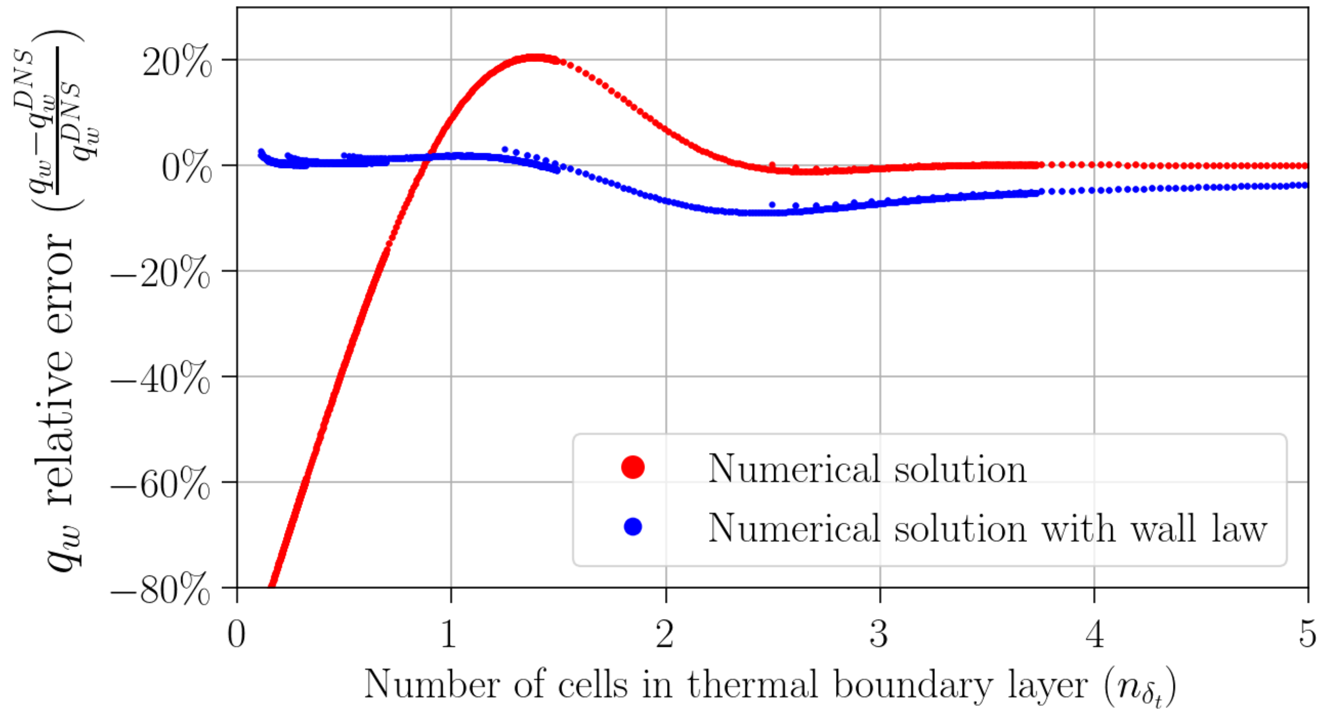
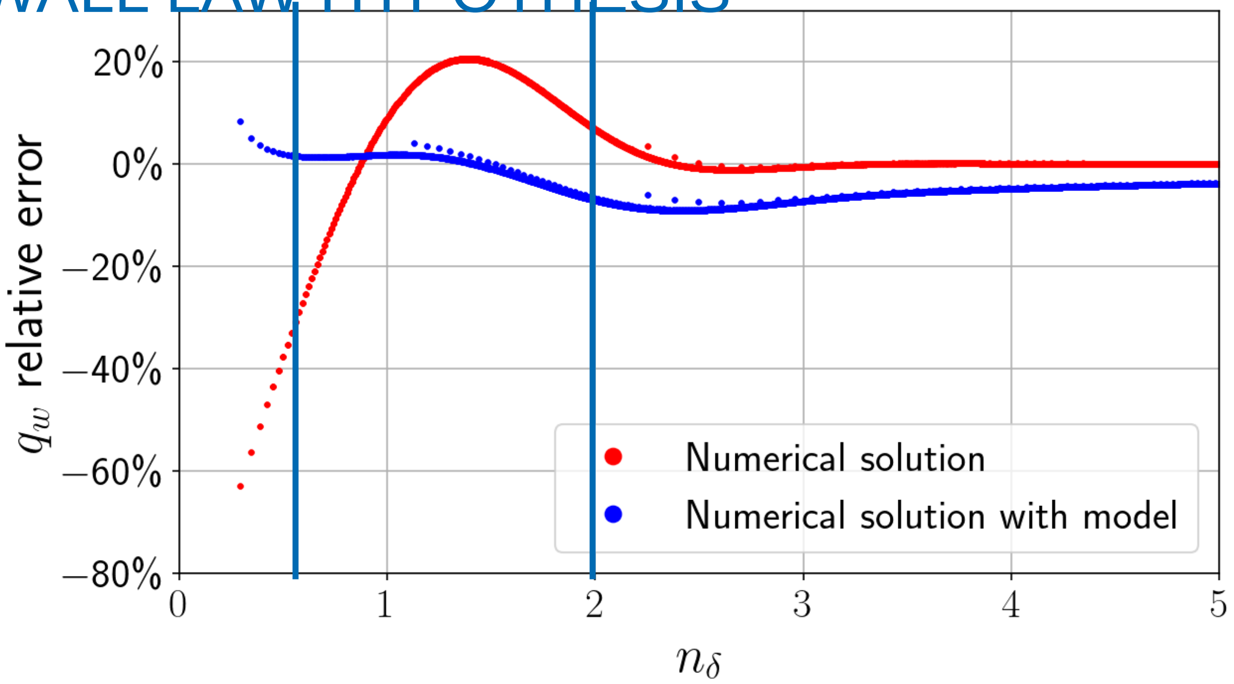


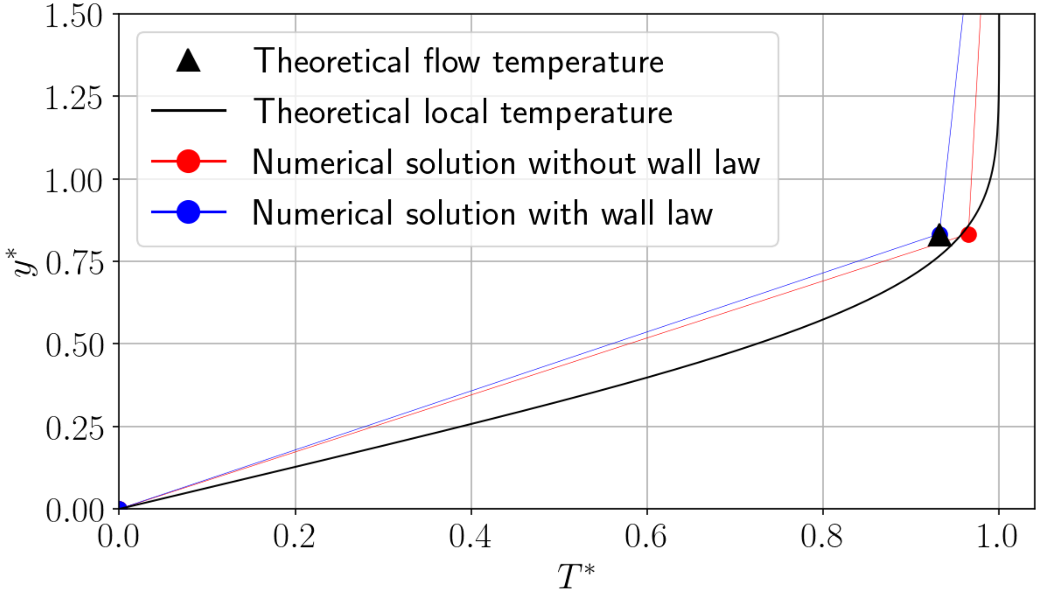
- Impact of the wall law on q_w prediction:
  - **Strong improvement for coarse mesh** ($n_{\delta t} < 1.5$)
  - **Error < 10% for more refined mesh**

SINGLE-PHASE FLOW VERIFYING WALL LAW HYPOTHESIS

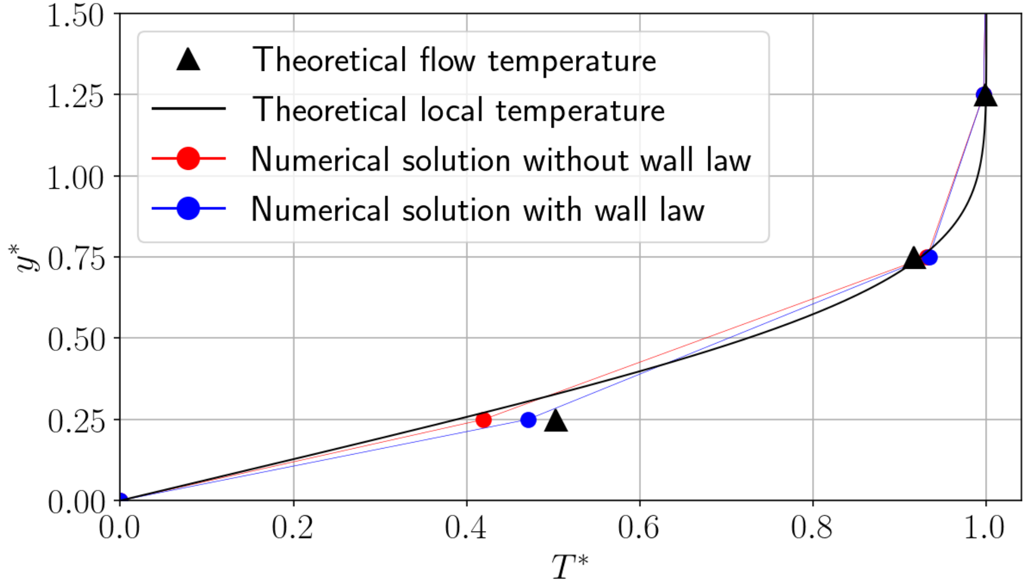
- **n_δ = 2**: error on T_1 leads to q_w error



Temperature profile for n_δ = 0.6



Temperature profile for n_δ = 2

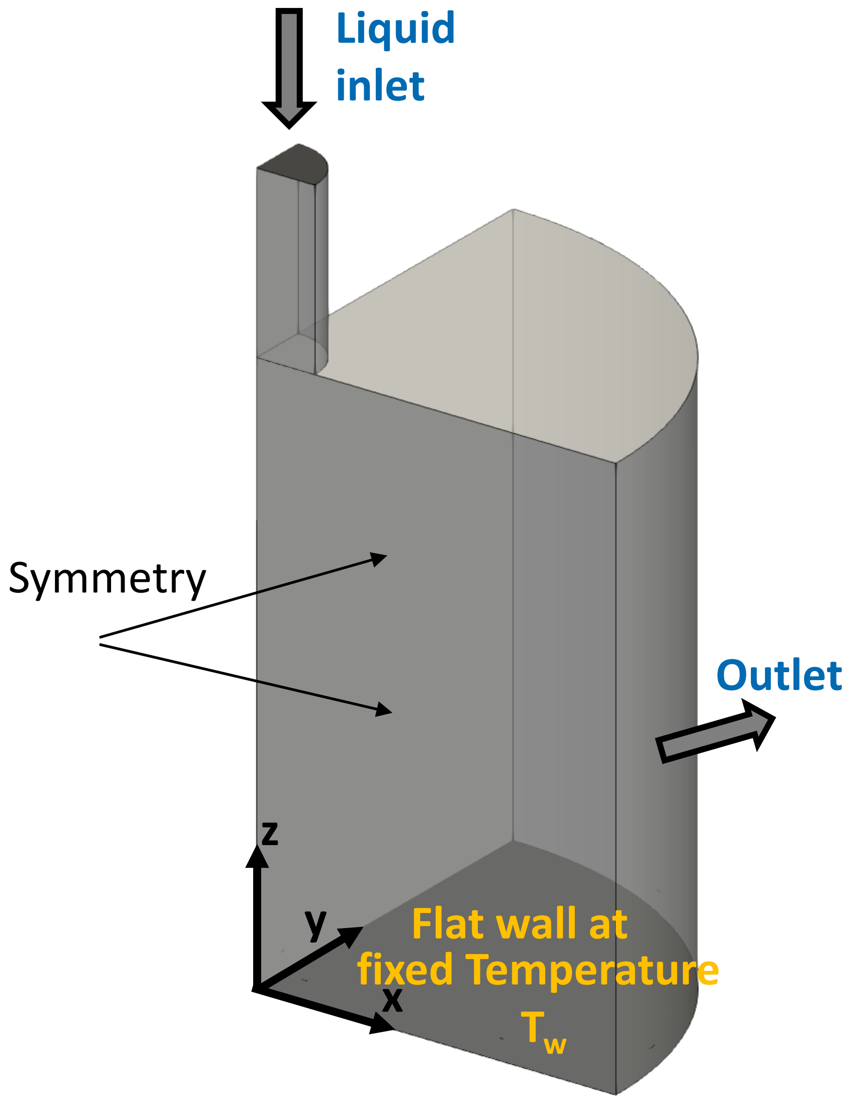


ASSESS THERMAL WALL LAW ON TWO-PHASE FLOW OUTSIDE THE WALL LAW HYPOTHESIS :

IMPINGING JET CONFIGURATION

IMPINGING JET CONFIGURATION FROM [1]
“POUBEAU ET AL. 2023”

- Modelling of a laminar impinging jet on a heated flat wall:
  - Using numerical setup from [1]
    - additional assumption of a uniform wall temperature (T_w)
    - VoF with HRIC scheme employed
    - validated with experimental data from [2]



- <u>Computational domain:</u>

[1] Poubeau et al. 2023 [2] Kekelia et al. 2015

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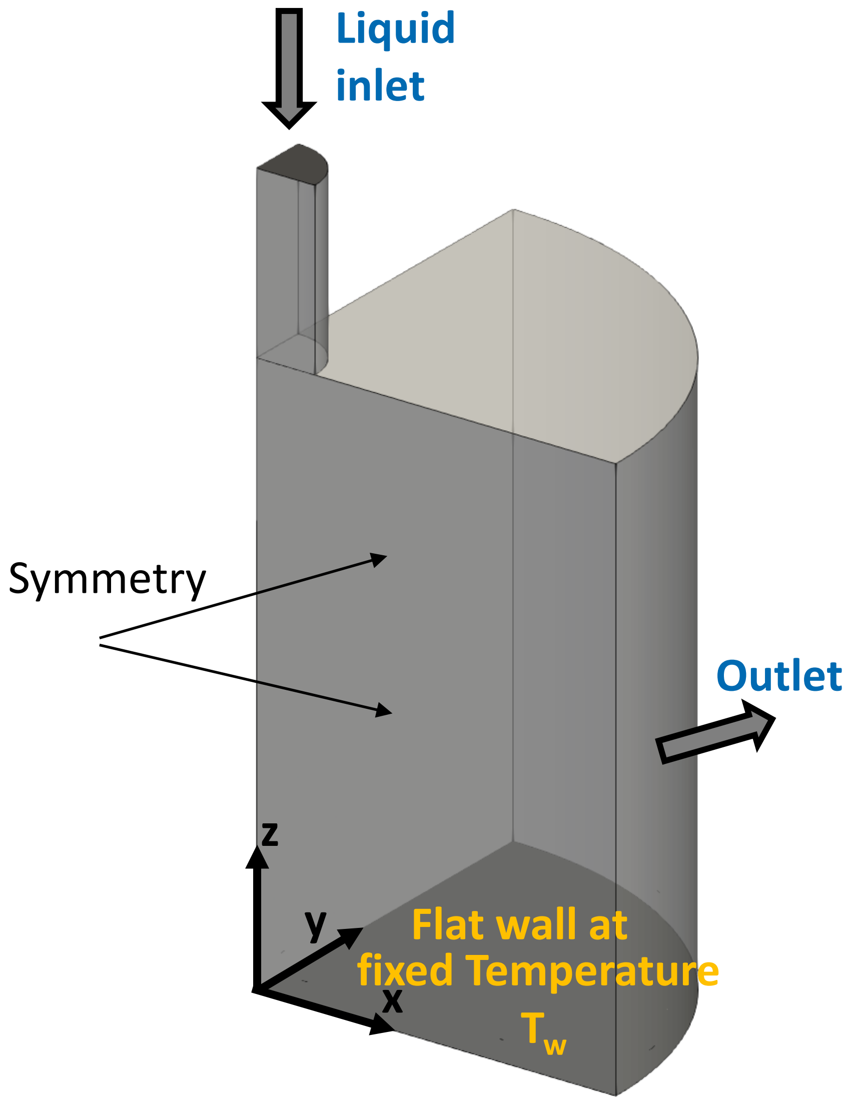
- Operating conditions and grids:
  - **Uniform viscosity assumption** $\mu \neq \mu(T)$
  - $Re_{jet} = \frac{u_{jet} d_{jet}}{\nu} = 900$
  - 2 Prandtl investigated:
    - $Pr = 160$
    - $Pr = 1000$
  - 2 meshes for each Prandtl
    - one thin mesh “DNS”
    - one under resolved mesh for δ_t



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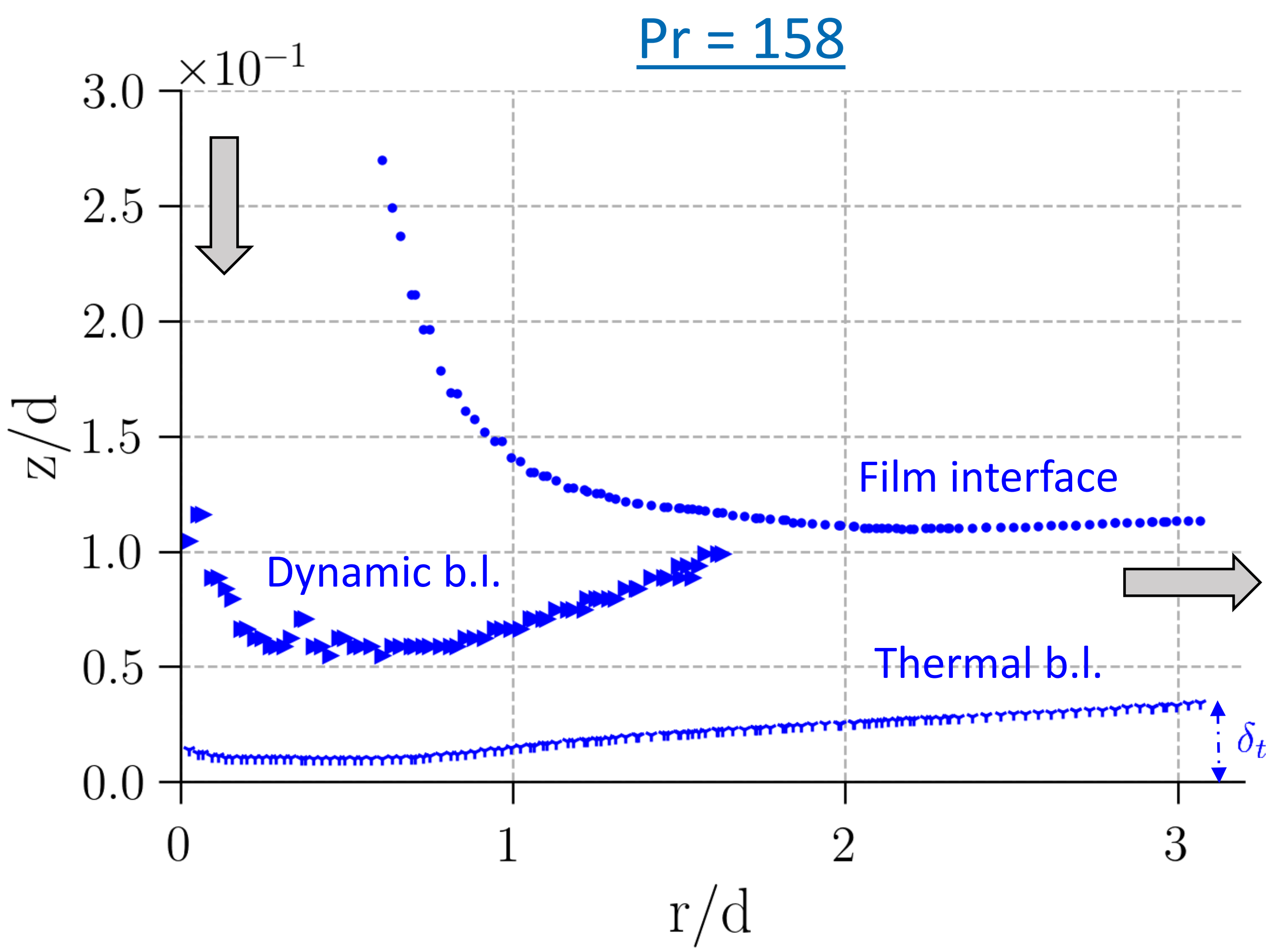
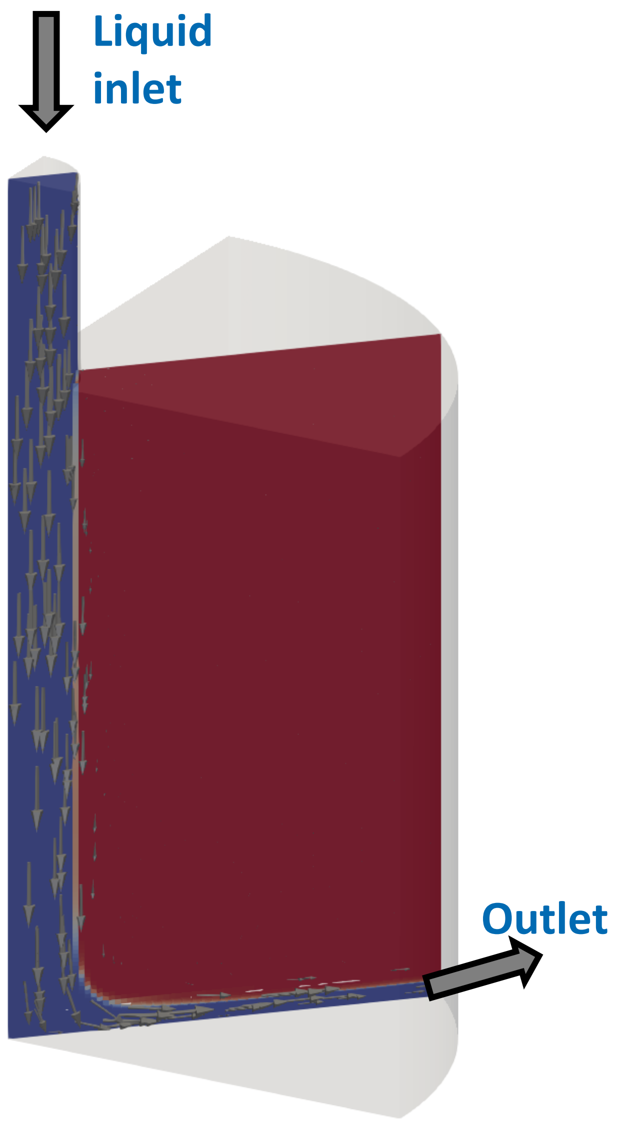
WALL LAW ASSESSMENT ON IMPINGING JET WITH **UNIFORM VISCOSITY**

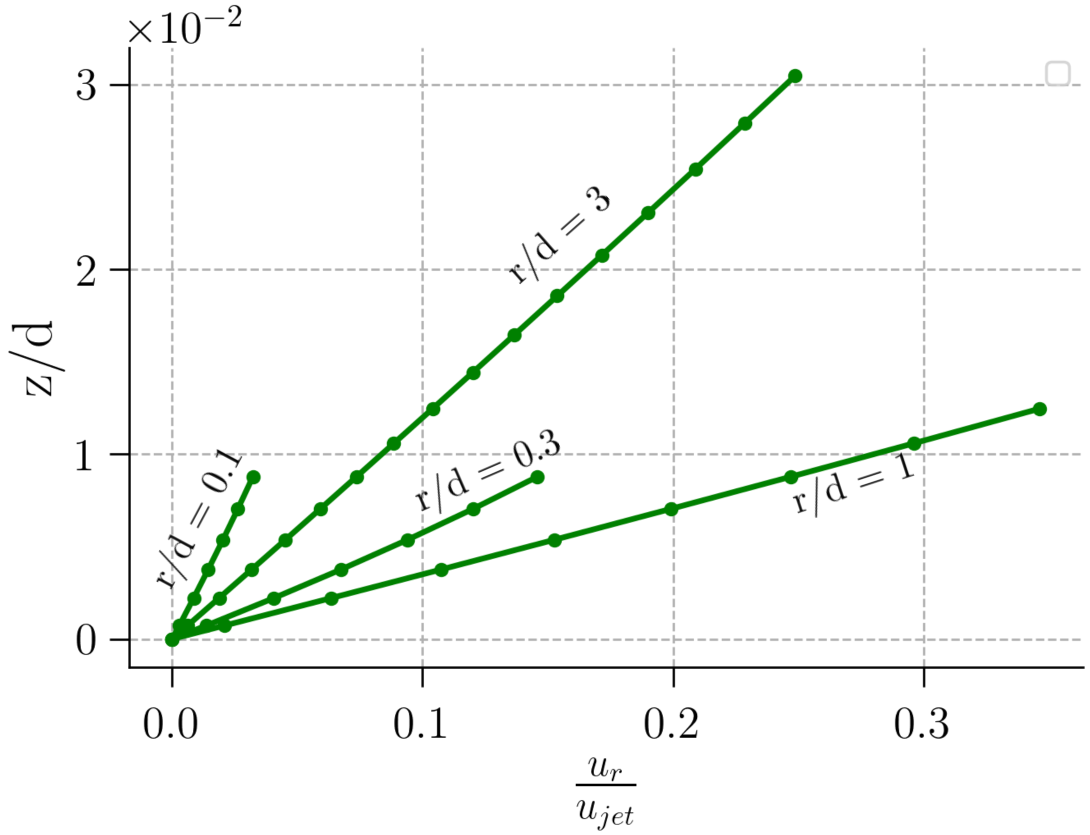




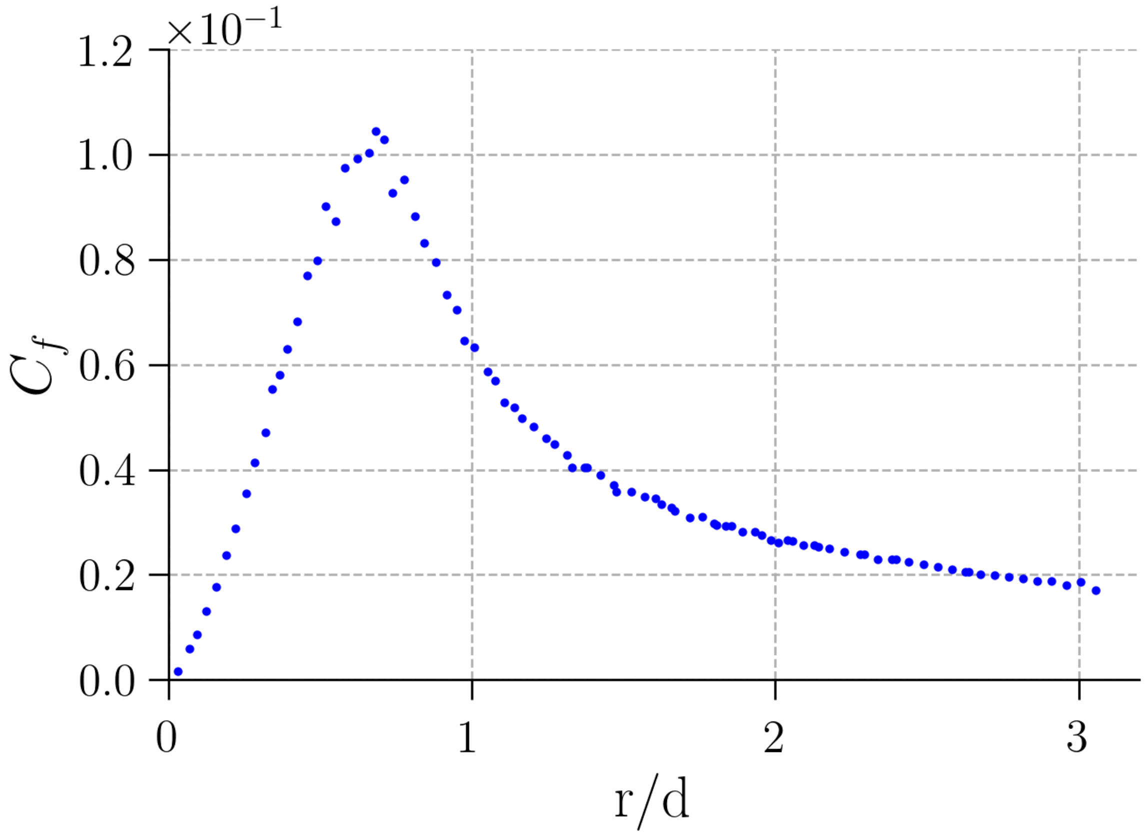
WALL LAW ASSESSMENT ON IMPINGING JET WITH **UNIFORM VISCOSITY**

- 2 wall law's assumptions not respected:
  1. Linear velocity profile
  2. Uniform shear stress over the wall

Velocity profile inside thermal boundary layer (Pr = 158)



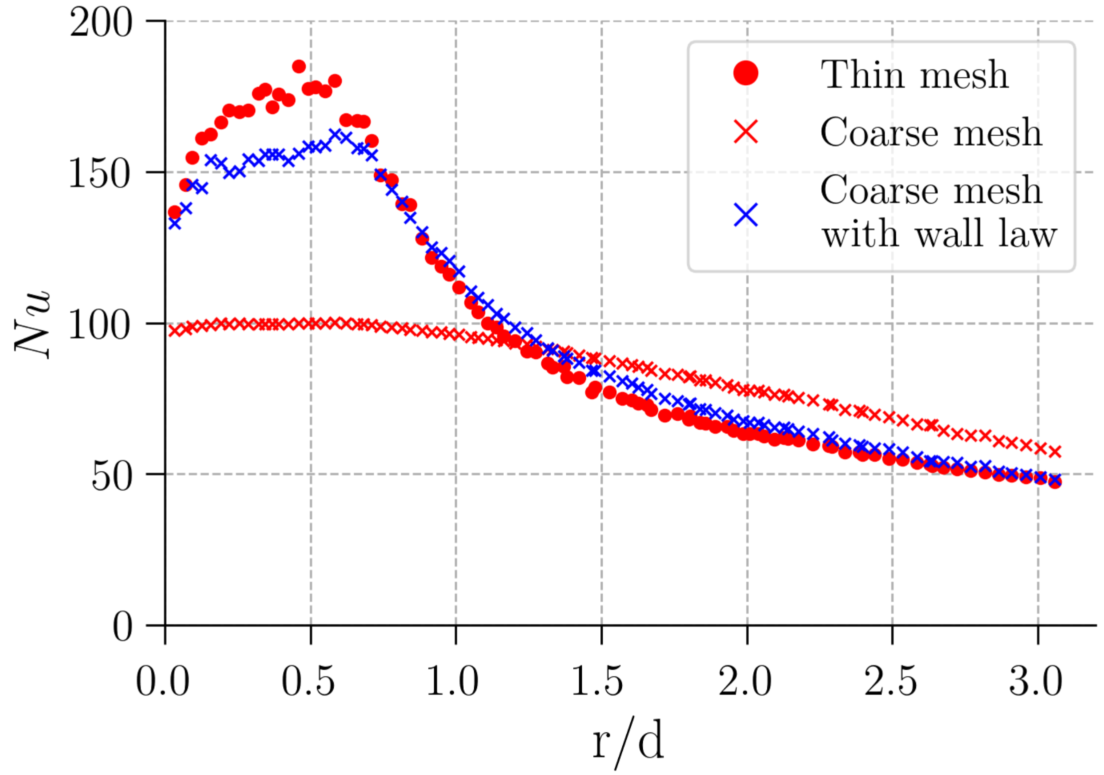
Wall shear stress



WALL LAW ASSESSMENT ON IMPINGING JET WITH UNIFORM VISCOSITY

- Significant improvement of the local wall heat flux prediction with the thermal wall law
  - Good prediction also in the stagnation zone (r/d < 1)

Pr = 158



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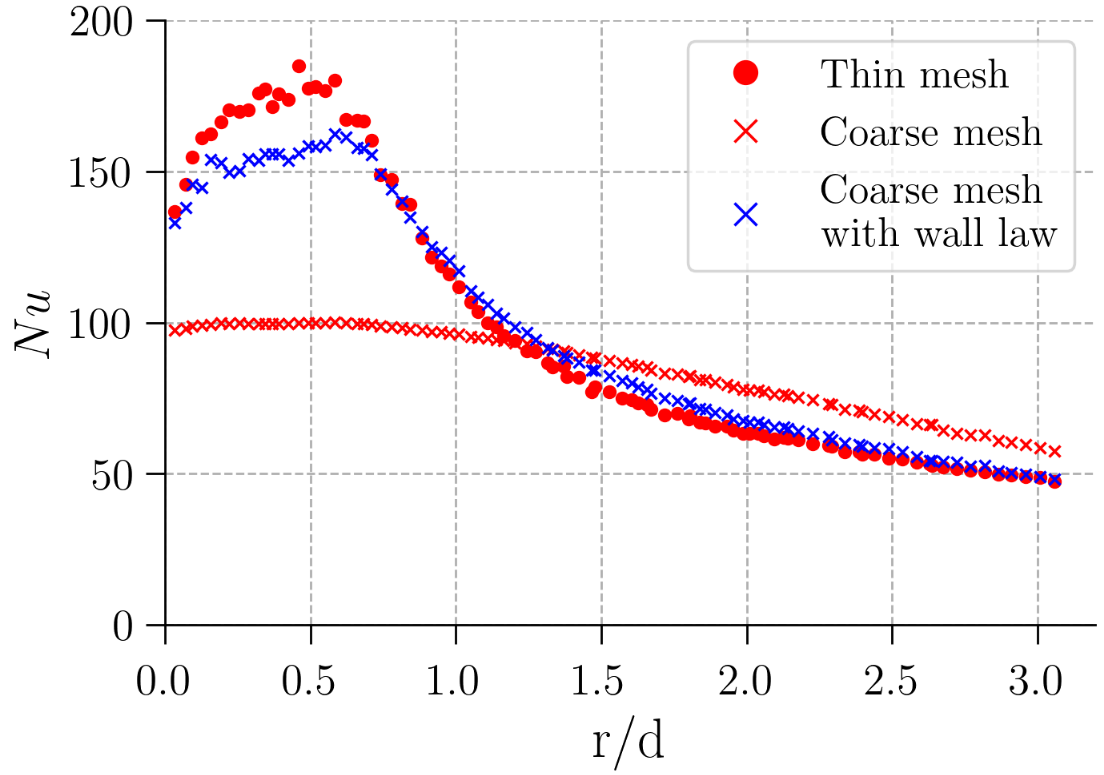
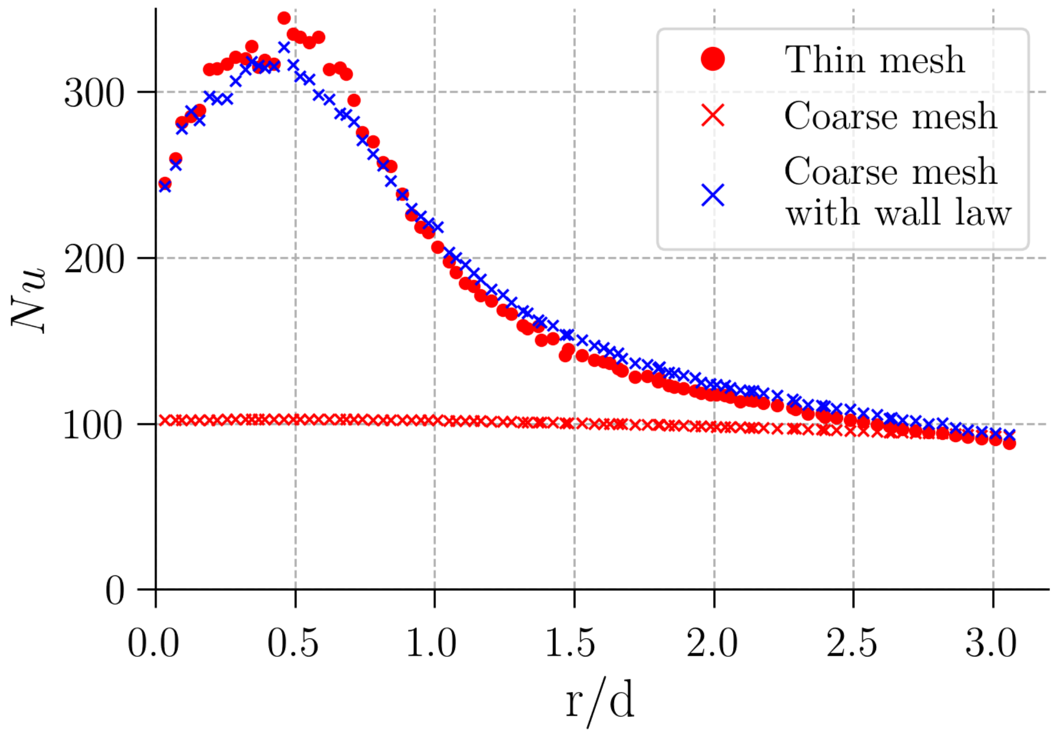


Pr = 1000



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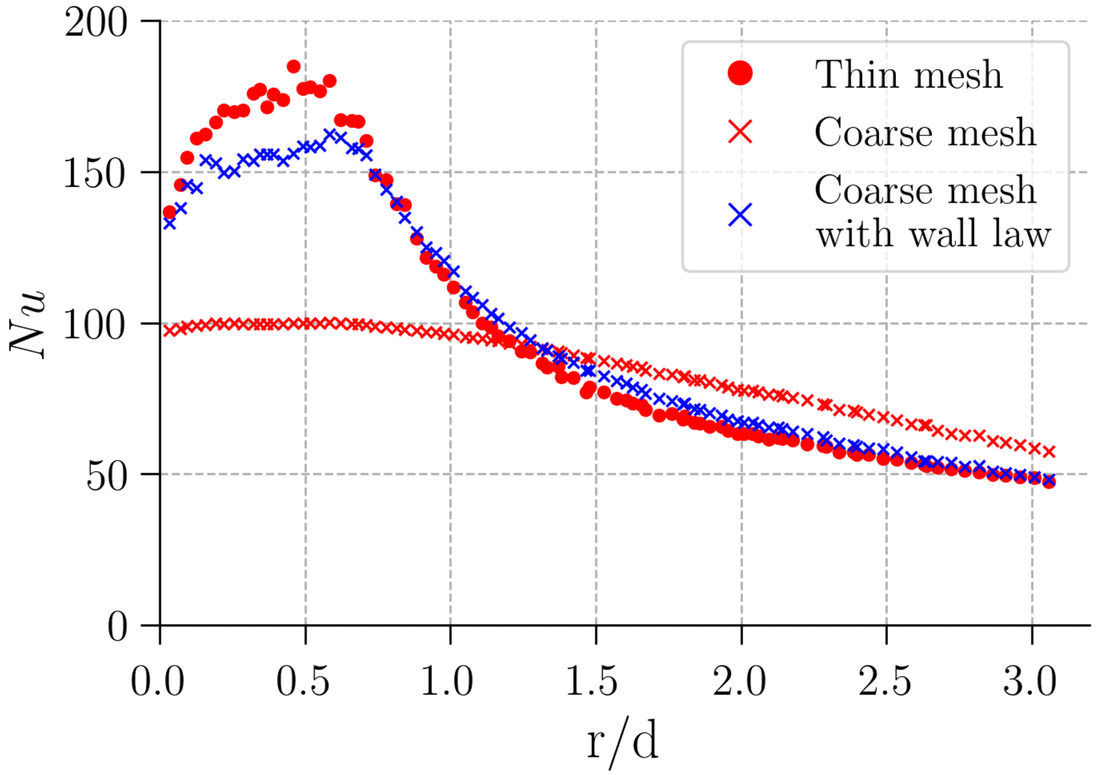
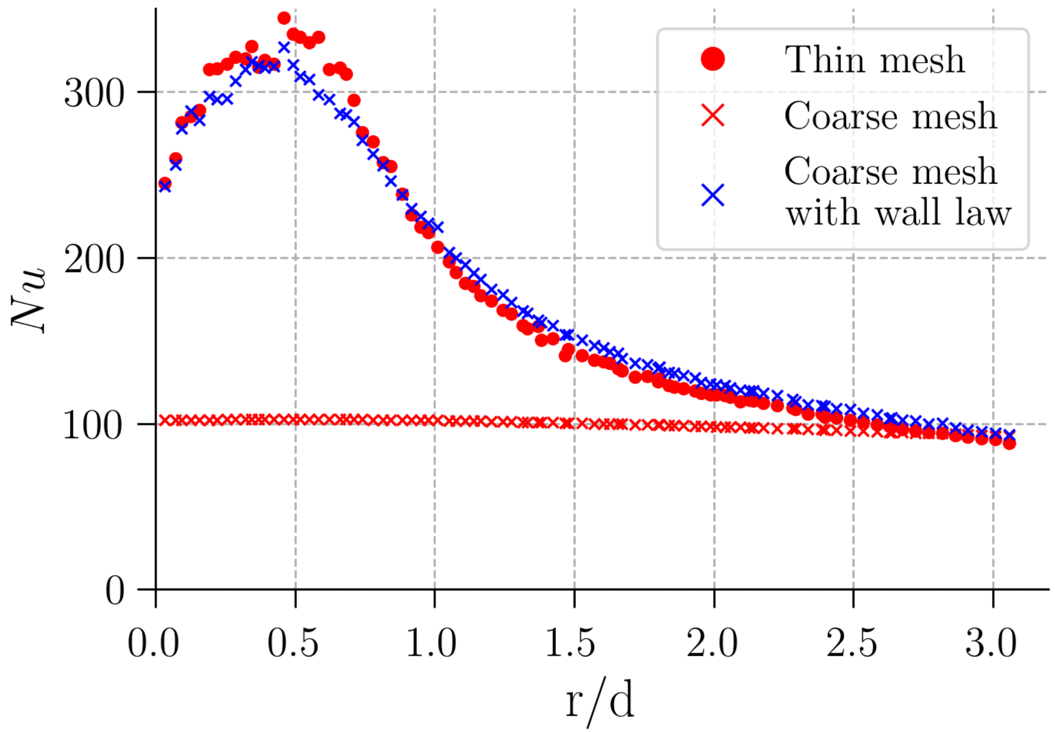
- Significant reduction in computational time for similar thermal wall heat flux prediction:
  - \- 70 % computational time (Pr = 158)
  - \- 90 % computational time (Pr = 1000)

Pr = 158



Pr = 1000



ASSESS THERMAL WALL LAW ON TWO-PHASE FLOW OUTSIDE THE WALL LAW HYPOTHESIS:

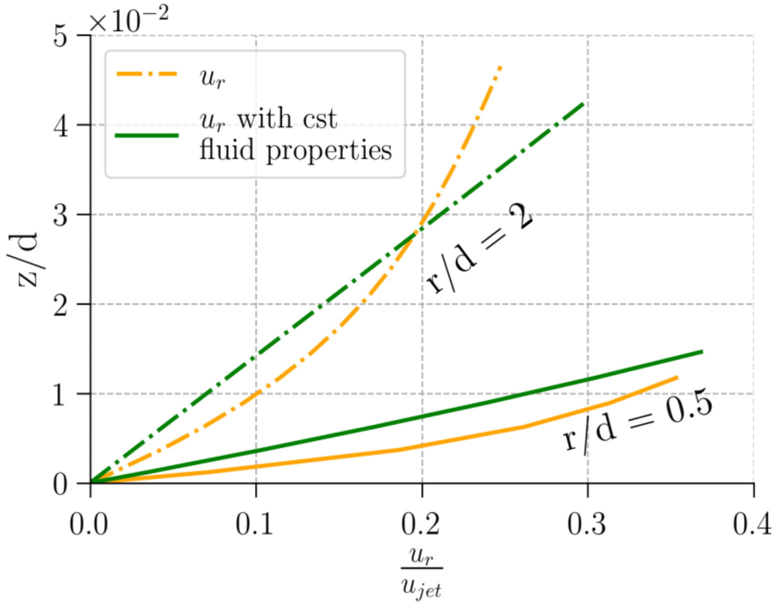
IMPINGING JET CONFIGURATION **WITH TEMPERATURE DEPENDENT VISCOSITY**

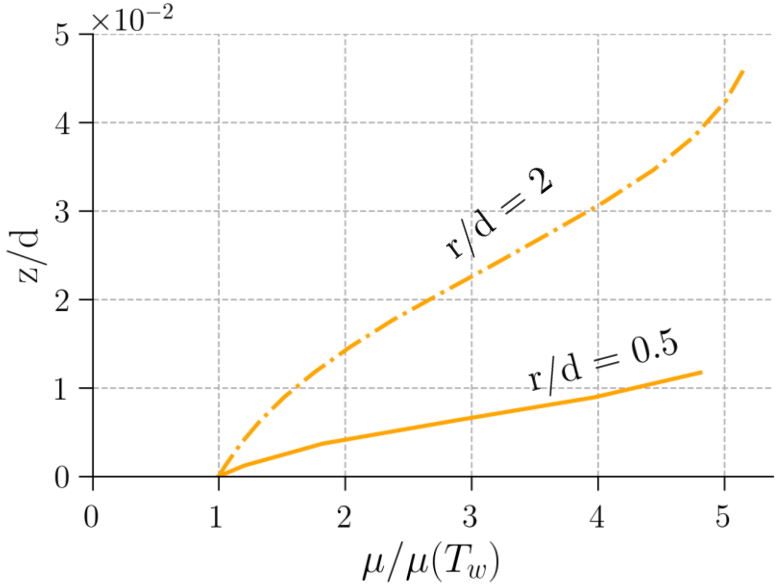
WALL LAW ASSESSMENT ON IMPINGING JET WITH **TEMPERATURE-DEPENDENT VISCOSITY**

- Real cases (Re = 320, Pr = 111) show velocity slope variation across the thermal b.l.
  - Sensitivity to dynamic viscosity variation across thermal b.l.

- Thin mesh across the thermal b.l. is necessary to correctly calculate the velocity field

- It is not possible to obtain satisfying results for coarse mesh with current thermal wall law.
  - The velocity field needs to be correctly calculated





CONCLUSIONS

- A thermal wall law was developed to address high Prandtl liquid, laminar flows.
  - Model based on Lévêque theory
  - Predict heat flux from first off-wall cells variables, imposing wall Temperature.

- Implemented in a CFD code (CONVERGE v3.2.4) using the finite volume method.

- First results:
  - Significant improvements for calculate wall heat flux with a coarse mesh assuming model assumptions are met:
    - With 0.5 cells in thermal b.l.: reduced q_w error from 40% to 1%.
    - q_w error less than 10 %
  - Application to impinging jet simulations with uniform viscosity :
    - significant reduction of wall heat flux error with a coarse mesh

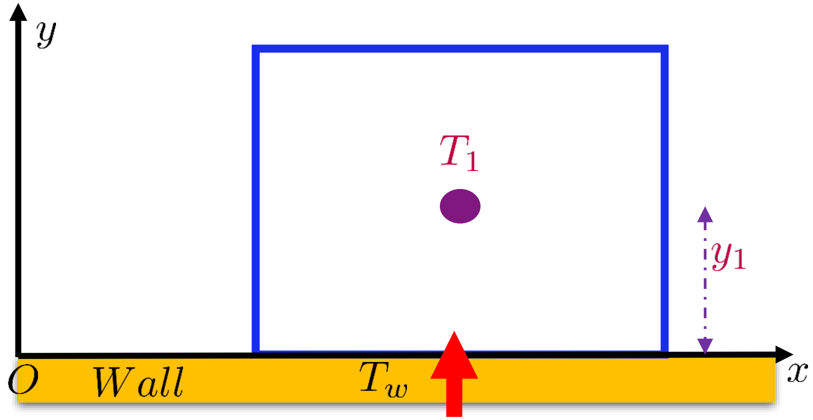
- Perspectives:
  - Reduction q_w error for thinner meshes by modify heat flux between first and second cell.
  - Test case with thermal b.l. inside non-linear profile (Blasius velocity profile)
  - Test case with a change in flow direction (reversal of velocity gradient)
  - Addressing jet impinging cases with temperature-dependent viscosity.
  - Addressing scenarios that are more representative of industrial applications.

jet impact simulation on realistic end-winding geometry [1]

[1] Poubeau et al. 2024

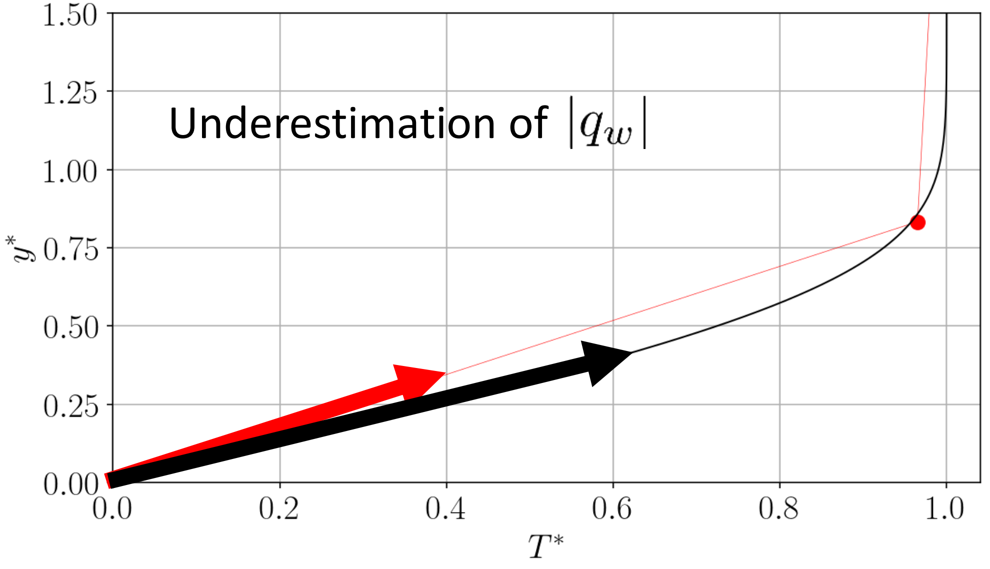
APPENDIX A: q_W CALCULATION METHOD IN CONVERGE

- q_w calculated with the Fick's law
- q_w error due to:
  - Linear approximation for small n_δ
  - Error on cell temperature for larger n_δ

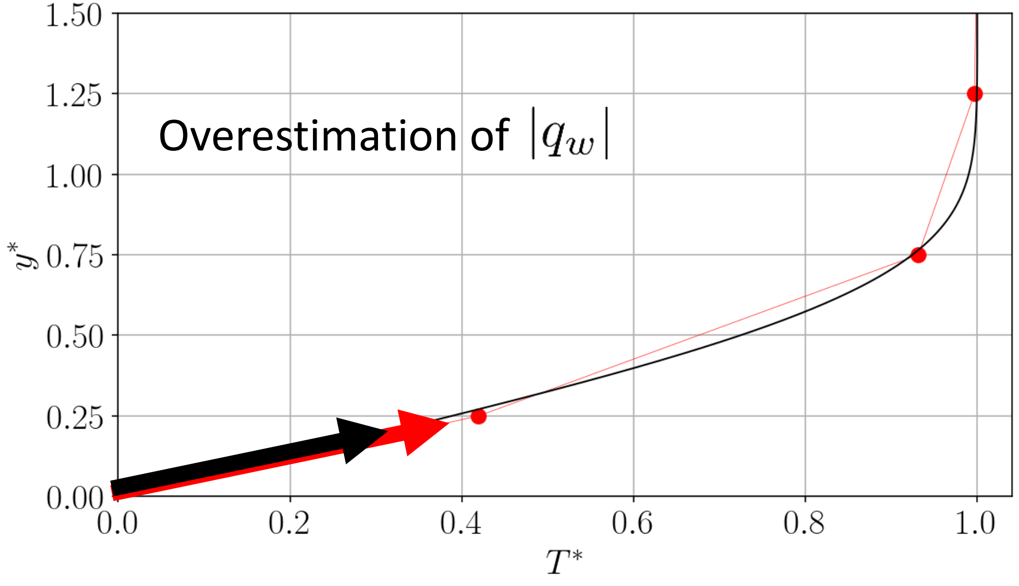


$$q_w = -k\frac{T_1 - T_w}{y_1}$$

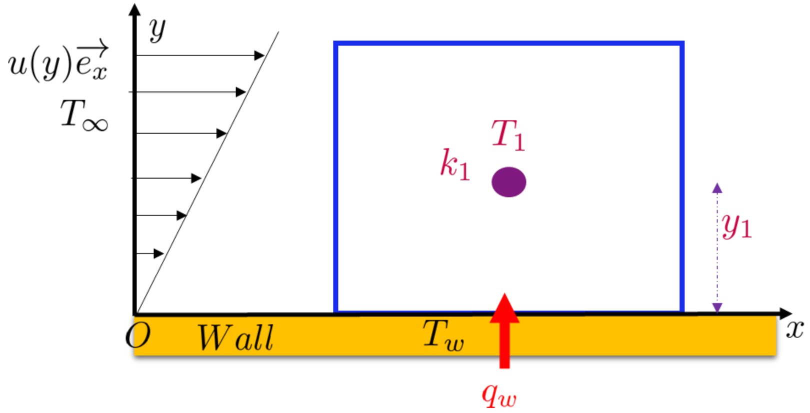
Temperature profile for $n_\delta = 0.6$



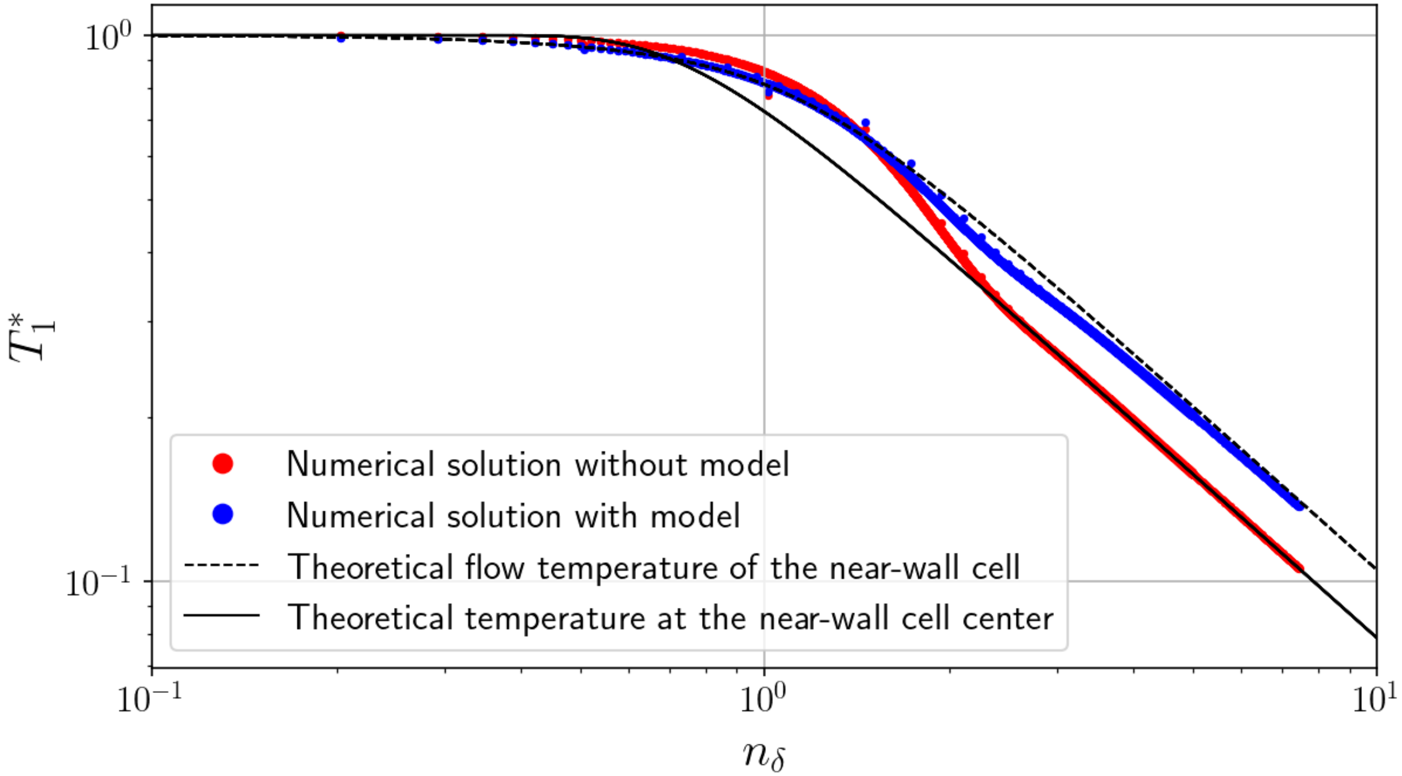
Temperature profile for $n_\delta = 2$



- Without model:
  - T_1 tends to be **equal to the local temperature** at the center of cell 1 for $n_\delta > 2$
- with model:
  - T_1 tends to be **equal to the flow temperature** of cell 1
  - q_w error comes from the error on T_1

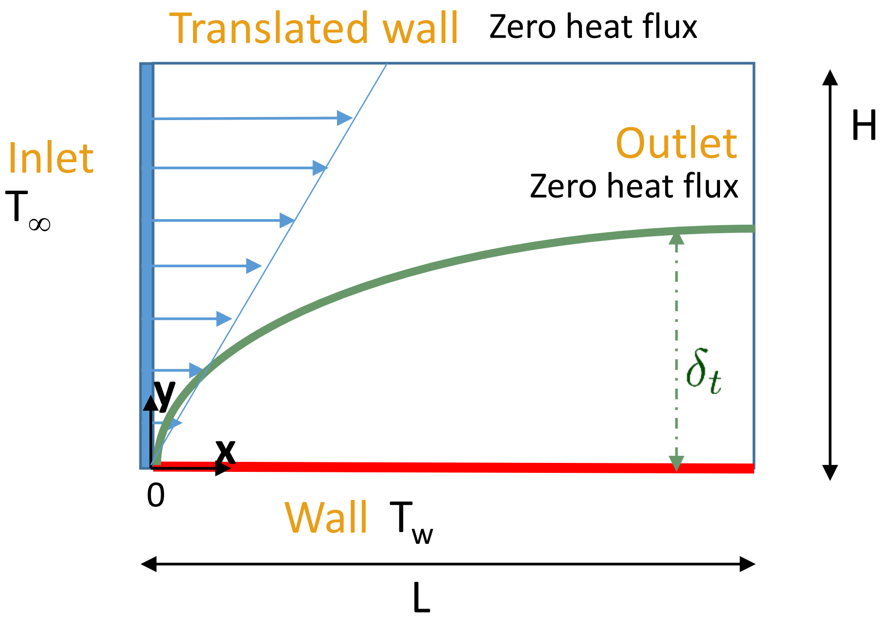


T_1* VS n_δ & comparison with theoretical temperatures

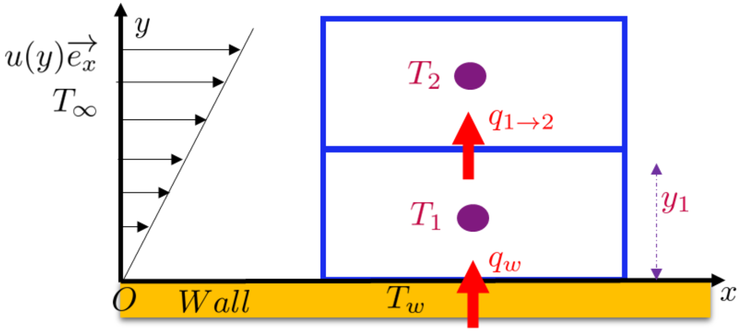


SINGLE-PHASE FLOW VERIFYING WALL LAW HYPOTHESIS

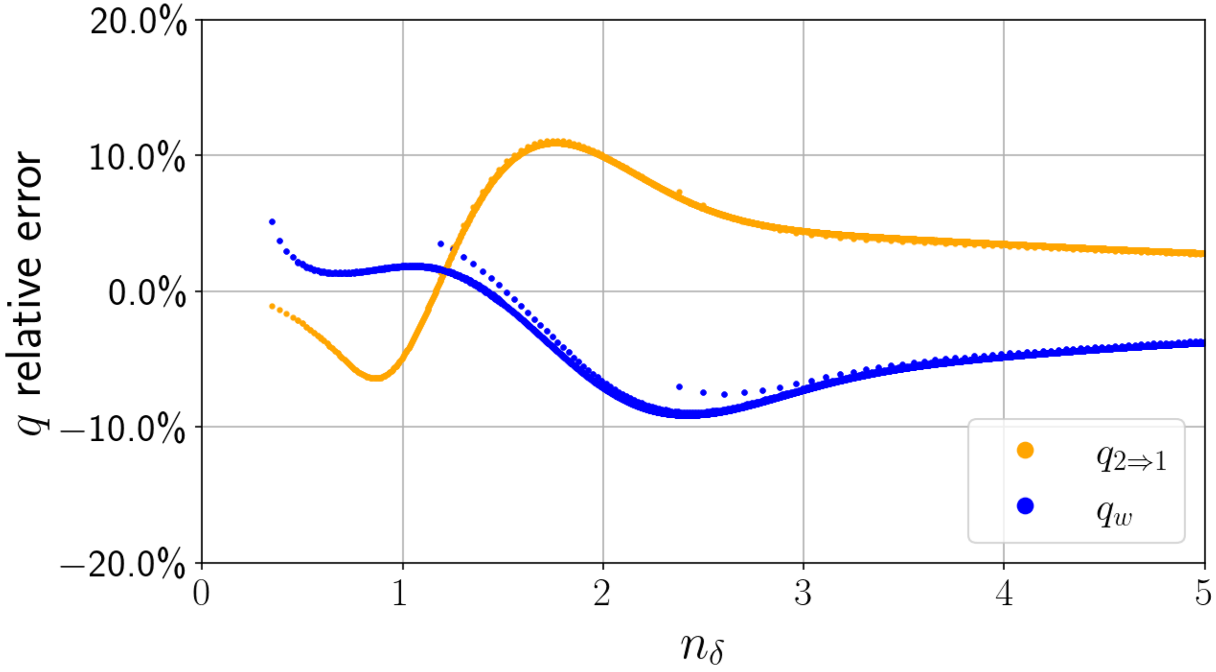
➢ Numerical set-up:



- Uniform cartesian mesh

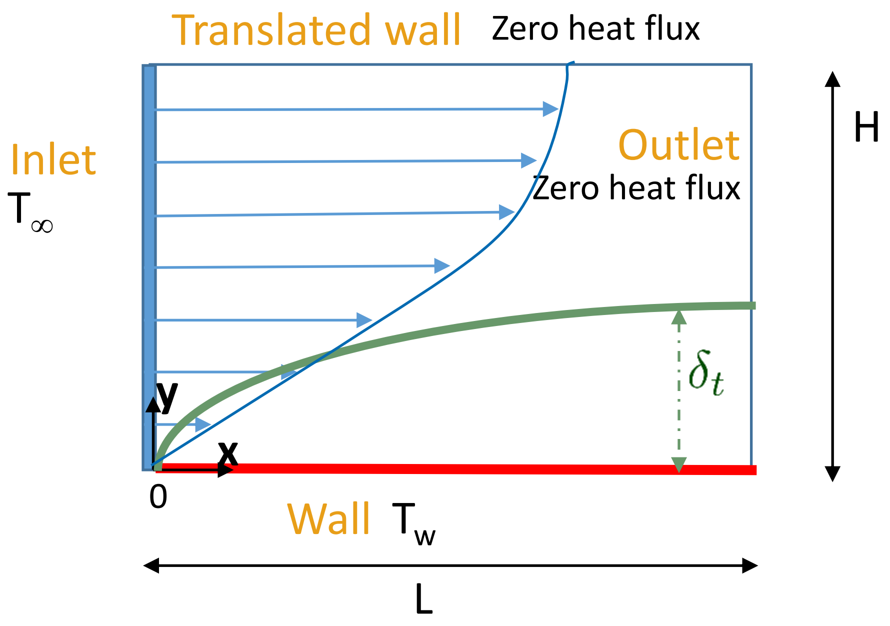


➢ Heat flux error with thermal wall law:

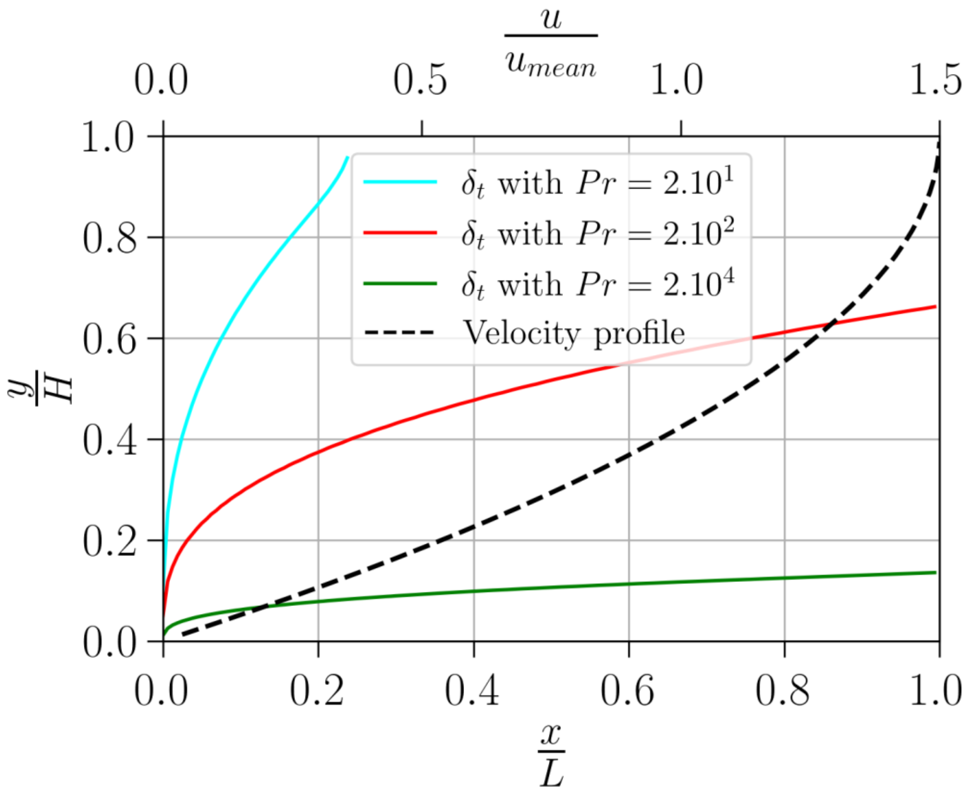


APPENDIX F1: ADDITIONS TO SINGLE-PHASE FLOW IN 2D POISEUILLE FLOW

➢ Numerical set-up:



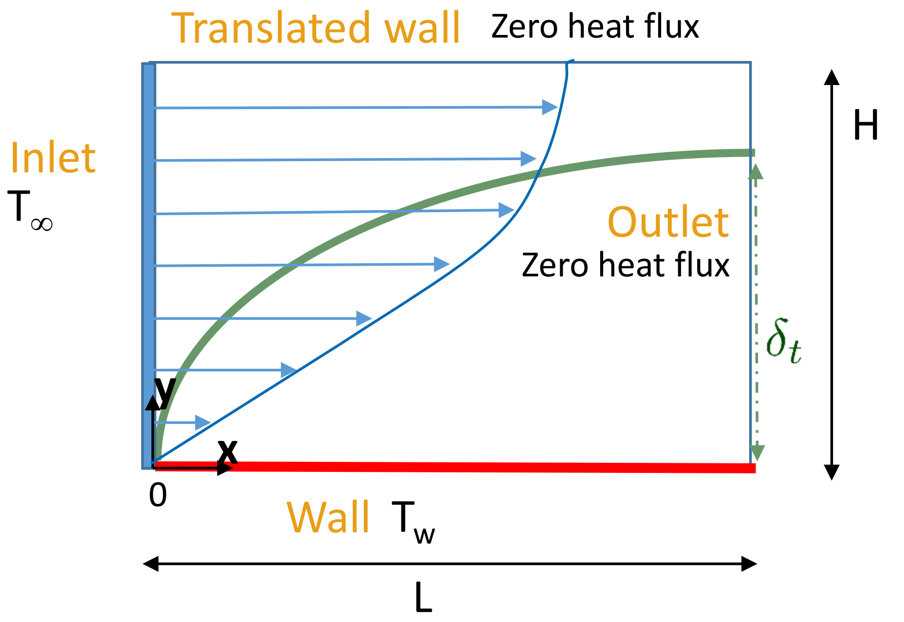
➢ Thermal boundary layer in Poiseuille velocity profile with Re = 50:



- "Mesh criteria" $\frac{2y_1}{H}$ is used to characterise the sensitivity of the wall law to the non-linearity of the velocity profile:
  - Higher mesh criteria -> higher impact of the non-linearity on the near wall cell temperature

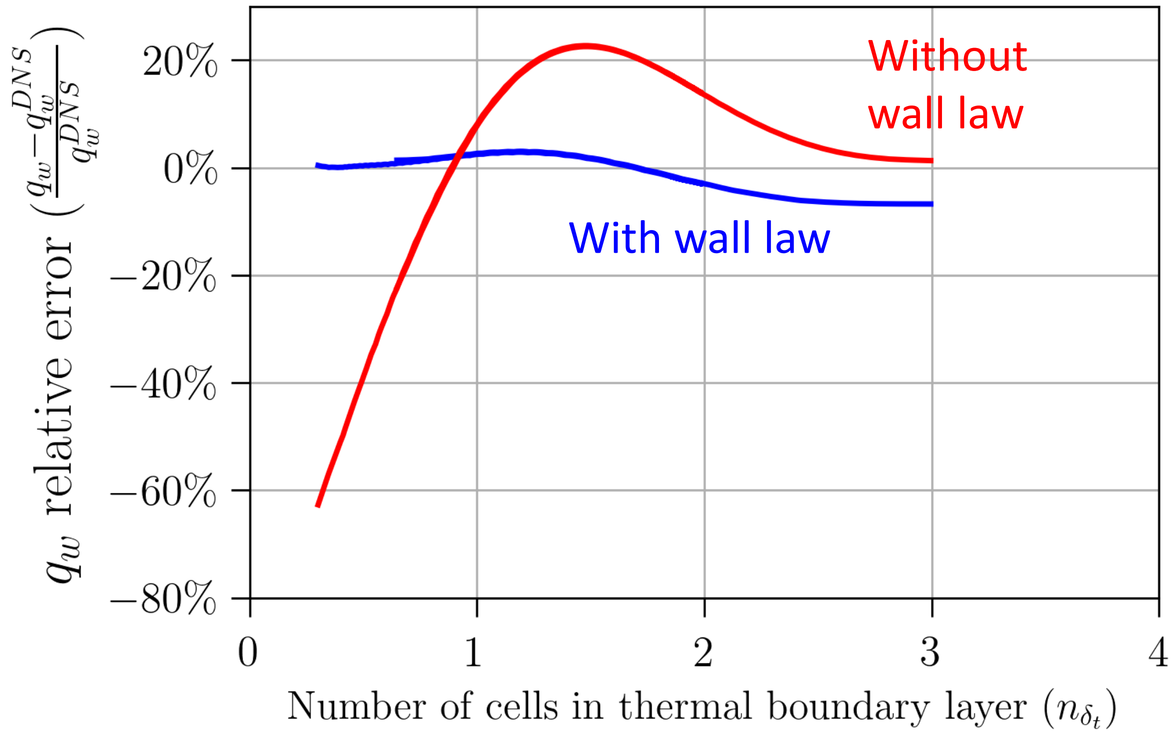
APPENDIX F1: ADDITIONS TO SINGLE-PHASE FLOW IN 2D POISEUILLE FLOW

➢ Numerical set-up:



- Uniform cartesian mesh

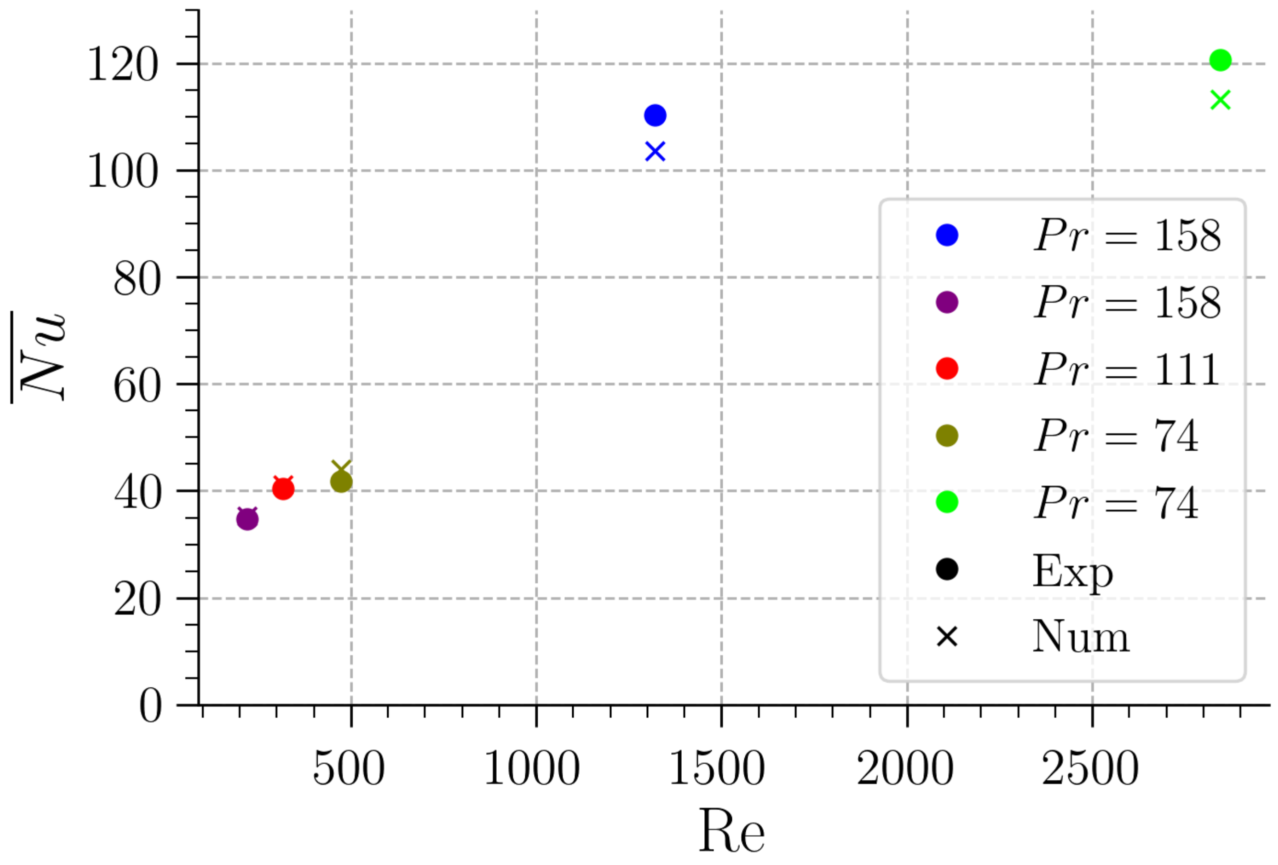
➢ Impact of a non-linear velocity profile on q_w prediction with 3 cells along Y direction

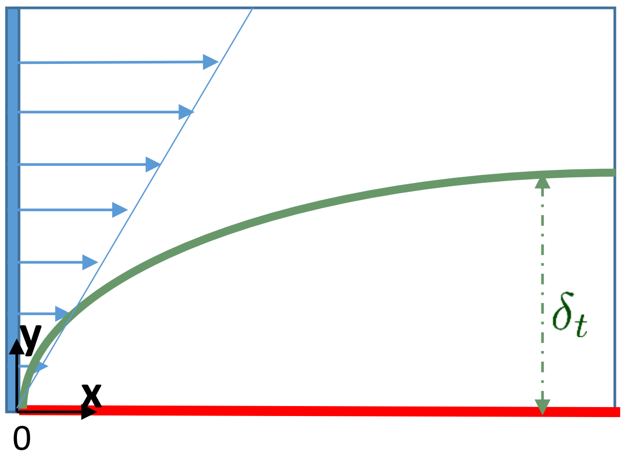


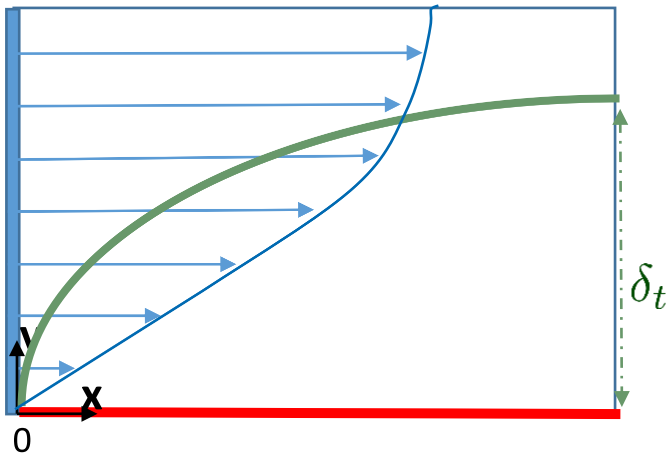
- **Wall law still relevant with Poiseuille profile**

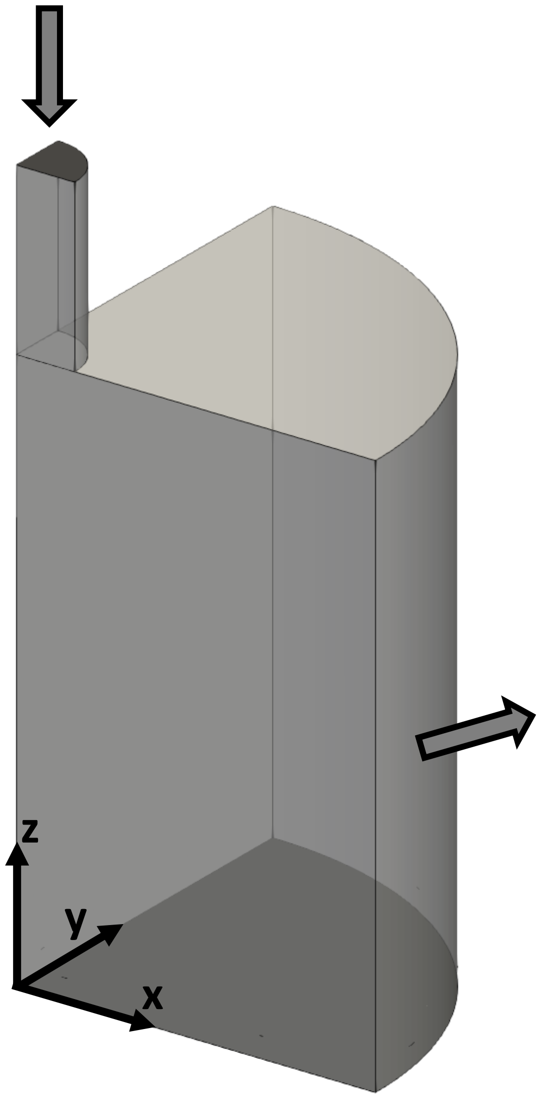
NUMERICAL SETUP VALIDATION

- Validation using mean Nusselt number over the wall
- Validation performed for different Re and Pr numbers (defined at the film temperature)
- Good consistency between numerical and experimental results




y
x
0
δ_t


y
x
0
δ_t


z
y
x

